6. Capacitance and Electric Field Permittivity

6.1. Purpose

The goals of this experiment are to cover the following:

1. Understand the behavior of a capacitor during charge-discharge cycling.
2. Show the relationship between capacitance of two parallel plates and their separation.
3. Measure the relative permittivity of three dielectrics.

6.2. Apparatus

parallel plate capacitor on translation mounting, $120k\Omega$ resistor, oscilloscope with two probes, signal generator, leads, 30cm ruler.

6.3. Introduction

Capacitors are used in many ways in electric and electronic circuits. These can be the storage of energy such as the starter capacitor in electric motors or high intensity lamps. They can be used to delay voltage changes when coupled with resistors. With an inductor and a capacitor, resonant circuits (oscillator) can be formed as well as bandpass circuits (tuning of radio and television signals). They can also be used to produce frequency dependent and independent voltage or current gain circuits.

A capacitor is essentially a pair of conductors that are separated by an insulator. Conductors allow the free movement of electrons while an insulator does not. When a voltage is applied across the capacitor, a charge of $+Q$ will build up on one plate, while a charge of $-Q$ will appear on the other plate. This results in the presence of an electric field between the plates. Although the insulator does not allow the flow of electrons, the molecules inside the material, especially for a dielectric, orient themselves to the electric field to create a surface charge.

Capacitance is given the unit of farad (F) after Michael Faraday. One farad is a huge amount of capacitance and common capacitors range from picofarads to millifarads. A capacitor has a capacitance of 1 farad if 1 coulomb of charge is deposited on the plates by a potential difference of 1 volt across the plates. Equation 6.1 shows the relationship between the voltage, charge and capacitance.

$$C = \frac{Q}{V} \tag{6.1}$$

where C is in farads, Q in coulombs and V in volts.
The value of the electric field between the plates is a function of the potential difference across the plates V in volts and the plate separation d in meters as shown in equation 6.2.

$$|\vec{E}| = \frac{V}{d} \tag{6.2}$$

The capacitance is found using equation 6.3.

$$C = \frac{\epsilon A}{d} \tag{6.3}$$

where ϵ is the permittivity of the material between the plates, A is the area of one of the plates and d is the separation between the plates. If a vacuum exists between the plates then $\epsilon = 8.85e - 12\frac{F}{m}$. A relative permittivity ϵ_r is often used which is the ratio of the permittivity of of the dielectric and the permittivity of free-space as per equation 6.4.

$$\epsilon_r = \frac{\epsilon}{\epsilon_0} \tag{6.4}$$

The relative permittivity of air has been measured to be $\epsilon_{r,air} = 1.0006$.

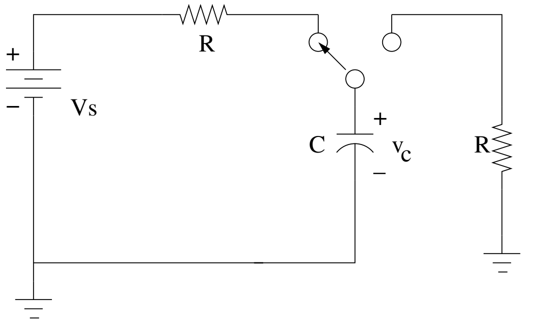

Figure 6.1.: Capacitor charge/discharge circuit with battery.

6.4. Capacitor Charge and Discharge Equations

Given the circuit shown in figure 6.1, consider that the capacitor has zero initial charge and the switch is closed at time t=0. Current will begin to flow through the resistor at a maximum rate and slowly decrease to zero when the capacitor is fully charged. The voltage across the plates will begin to rise and eventually equal the source voltage V_S. Equation 6.5 shows the mathematical relationship of the capacitor voltage $v_C(t)$ when charging the capacitor.

$$v_c(t) = V_s\left(1 - e^{\frac{-t}{RC}}\right) \tag{6.5}$$

Figure 6.2 is a plot of this equation. Of special interest is when $t = RC$ where R is in Ohms and C in Farads, v_c will be $0.632 \cdot V_s$. Table 6.1 shows a few other values.

t	$1 - e^{\frac{-t}{RC}}$
$1 \cdot RC$	0.632
$2 \cdot RC$	0.865
$3 \cdot RC$	0.950
$4 \cdot RC$	0.981

Table 6.1.: Charging values at certain times $t = n \cdot RC$

When the switch in figure 6.1 is closed to the right, the capacitor will start to discharge through the resistor that is tied to ground. Equation 6.6 describes the voltage v_C as a function of time t.

$$v_c(t) = V_s e^{\frac{-t}{RC}} \tag{6.6}$$

Figure 6.3 shows the discharge equation.

Rather than writing RC, τ is used and called the characteristic time.

$$\tau = RC$$

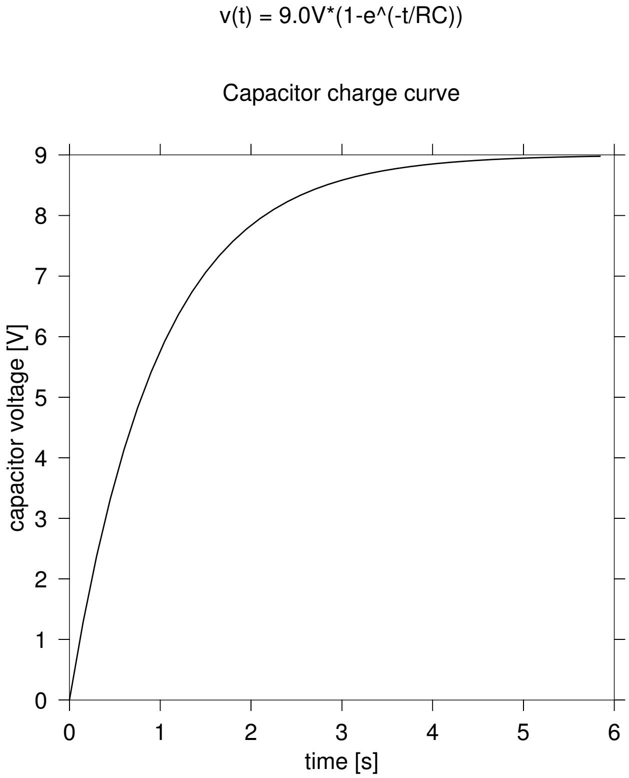

Figure 6.2.: Capacitor charge curve

The experiment at hand will use these equations to arrive at a value for the capacitance of a parallel plate apparatus and the dielectric constant of the three materials.

6.5. The Variable Capacitor Apparatus

Two circular parallel plates are mounted on a small bench so that plate separation may be adjusted. Binding posts are provided for connection to each plate. A millimeter scale allows the experimenter to measure and change this separation from approximately 1.5mm to 134mm.

The plates are to be parallel to each other throughout the experiment. This is especially critical for small separations. The space between the plates can also be filled with different materials to act as a dielectric. Figure 6.4 shows the apparatus.

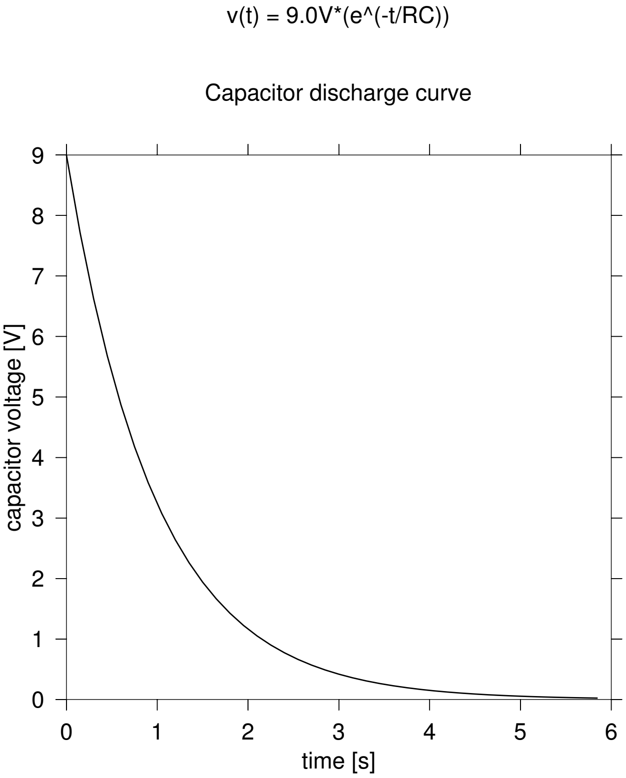

Figure 6.3.: Capacitor discharge curve

6.5.1. Calculation

Using equation 6.6, solve for C given the following conditions. These conditions are the method you will use to determine a value for C in the next stage of the experiment. You will be able to easily measure the time between the 90% and 10% voltage levels of the capacitor as it discharges.

1. Solve for C assuming that you have t_1 and t_2 which are the 90% and 10% charge levels present across the capacitor C.

6.5.2. Oscilloscope settings

The following settings have yielded good results.

- For the oscilloscope triggering, use external triggering, DC trigger coupling (no HF rejection). For the probes, use ×10[1].
- Use DC channel coupling for both inputs.

6.5.3. Plate Alignment

It is recommended that the plates be made as parallel to each other as possible. Check for this condition by moving the plates to minimum separation and check that contact is being made at the three plastic stops which keep the plates from touching. Adjustment is made using the three Allen key screws located on the back of the fixed plate. This will allow you to get the two plates approximately parallel.

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[1] Note that the compensation has adjustment has no affect if the probe is set to ×1. Using ×1 will reveal some loading affects on the capacitor voltage amplitude which will require compensation using the amplitude of the signal generator.

Figure 6.4.: Capacitor Apparatus

6.5.4. EXPERIMENT: Capacitance as a function of Plate Separation

The following instructions will get you close to being able to take measurements. Further tuning will be necessary.

1. Construct the circuit shown in figure 6.5.
2. Use channel one to monitor the signal generator's output and channel two to view the voltage across the capacitor. External triggering can be helpful when using an analog scope.
3. To begin, the following settings will give you a nearly usable trace which must be further tuned. Set the signal generator to provide a square wave. Set an amplitude of $6V_{PP}$ with a frequency of about

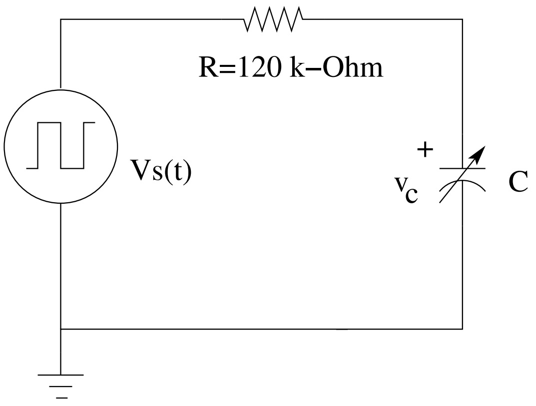

Figure 6.5.: Charge-Discharge circuit

$3kHz$ to $5kHz$. You will have to fine-tune both amplitude and frequency from here[2].

4. Set the Y cursors: BY (dashed) to +2.4V and AY (solid) -2.4V. This represents the *10%* and *90%* levels.

5. Your goal is to get a trace similar to that shown in figure 6.2 or 6.3 with the *time* between 90% and 10% spread out as much as possible on the oscilloscope display. Spreading out this waveform horizontally as much as possible will lead to most accurate results.

    a) First insure you have the waveform of interest going from *0%* to *100%*, that is -3V to +3V. This can be done using a combination of amplitude setting on the signal generator and the oscilloscope's off calibration dial. Adjust the variable (or "off-calibration") gain control for channel two (the little knob on the inside of the $\frac{volts}{cm}$ dial for each channel) to fit the top and the bottom of the waveform onto the 0 and 100 percent levels respectively. Adjustment of the trace position and the amplitude of the generator output can help.

6. Now use the X cursors (the dashed one is the B cursor) to measure this decay time (or charge time, either will do) and use the formula from your calculation in subsection 6.5.1 to solve for C. Repeat for following values of plate separation in mm: 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 14, 18, 20, 25, 30, 35, 40, 50, 60, 70. Keep an eye on where the trace is coming down through the 90% level. Adjustment of the horizontal position is necessary.

7. Plot a graph of capacitance[3] as a function of the inverse of plate separation *d*. Explain why the y-intercept is not zero as predicted by the equation.

8. Measure the plate diameter. Calculate the area A of a plate. Use this with your graph's slope to get a value for the permittivity of air.

9. Insert one of the test sheet materials between the plates and bring the plates into contact with it. Measure the time difference as above when the material is present and when it is not present.

Question

1. Does the materials presence in step 9 above increase or decrease the capacitance? Explain how your measurement can be used to determine this.

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[2] There should be zero DC offset, although this lab can be executed with one present.

[3] using the formula from subsection 6.5.1