4. Coulomb's Law

4.1. Pre-Lab Questions

1. What is the minimum current generally accepted to cause a human fatality?
2. How much resistance does a human body present to a voltage source when the skin is a) dry and b) wet?
3. For the case of wet skin coming into contact with a voltage source, how much voltage difference is required to produce the current from question 1?
4. If a $200M\Omega$ resistor is placed in series with the probe, what is the current flow produced when the power supply is set to $6000V_{DC}$ and the two leads placed in contact with one another?

4.2. Precautions

The electrostatic force on the two spheres is created by touching a probe attached to a High Voltage (HV) power supply to the spheres. Voltages of up to $6000V_{DC}$ are used which could easily produce a fatal current flow in a person. However the equipment is designed[1] to eliminate this risk. Regardless of this, a healthy respect for electricity is to be observed. The following questions must be answered before the lab is started.

4.3. Purpose

1. Determine the relationship between electrostatic force and charge separation.

4.4. Theory

Coulomb's Law states that the electrostatic force between two charges q (made up of charges q_i and q') can be expressed mathematically as

$$\vec{F} \propto \sum_{i}^{n} \frac{q_i q'}{r_i^2} \hat{r_i} \tag{4.1}$$

where $\hat{r_i}$ is the unit vector drawn from q' to q_i. The formula results from experiments done by 1785 by Coulomb and earlier by Priestley in 1767. A set of ideas is expressed by this formula as follows.

1. There is a quantity q called electric charge which may exist on or in matter. Its presence on two or more bodies is made evident by forces of attraction or repulsion between the bodies containing the charge.
2. There are two kinds of electric charge. The force between two like charges is repulsive and acts along the line joining them. Unlike charges have an attractive force between them. The charges are arbitrarily called positive and negative.

[1] Current is limited to a safe level by the power supply's internal resistance

3. It is implied that charge is conserved. If charge q_1 and q_2 are brought together, the total charge is $q = q_1 + q_2$. If we combine equal charges of opposite sign, then the net charge is zero. Further, this implies that for a particular body, we need only consider the net charge, or the excess of one charge (say negative) over the other (say positive).

4. The force between a pair of charges is inversely proportional to the square of the distance between them. This also means that there is a single distance that represents the separation, so the charges are considered to be points in size.

5. To calculate the force between charges, you *multiply* the *quantity* of the first charge by the *quantity* of the second charge.

6. The total force is computed by adding the separate force vectors. This implies that the force created between any two charges is unaffected by the presence of other charges. This follows the principle of *superposition.*

In order to calculate the force $\vec{F_1}$ in newtons, a constant of proportionality is needed, so for two charges q_1 and q_2, equation 4.1 becomes:

$$\vec{F_1} = K \frac{q_1 q_2}{r_{12}^2} \hat{r} \tag{4.2}$$

where it is convenient to write $K = \frac{1}{4\pi\epsilon_0}$ when the two charges are in vacuum. $\hat{r}_{12}$ is the unit vector draw in the direction *from* q_2 *to* q_1.

$$\hat{r} = \frac{\vec{r}_{12}}{|\vec{r}_{12}|}$$

The permittivity of free-space ϵ_0 has a value of $8.85 \times 10^{-12} \frac{coulombs^2}{newton-m^2} = 8.85 \times 10^{-12} \frac{F}{m^{-1}}$. The force is reduced for any medium other than vacuum and the permittivity constant becomes ϵ.

Permittivity is defined as the ratio the electric displacement $\vec{D}$[2] in a medium to the intensity of the electric field producing it.

$$\epsilon = \frac{|\vec{D}|}{|\vec{E}|}$$

It is important for electrical insulators used as dielectrics. It is a measure of the resistance that is encountered when forming an electric field in a medium.

Permittivity relates to a material's ability to transmit or "permit" an electric field. In a substance ϵ is reduced by movement of the electrons of the matter. The matter becomes polarized and this cancels out some of the applied field.

The electric field $\vec{E}$ is defined a region in which an electric charge experiences a force usually because of the presence of other charges. It has units of N/C. The electric field strength at any point in an electric field is defined as the force per unit charge experienced by a small charge placed at that point.

For the force on a charge q_0 due to a number of charges, we may write

$$\vec{F_0} = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_0 q_i}{r_i^2} \hat{r}_i$$

A simple modification can be made to handle the situation where the charge is distributed over a region instead of being concentrated at a particular point. To calculate the force on q_0 near a continuous charge

[2] $\overrightarrow{D}$ is the amount of charge displaced across a layer of a conductor of unit area placed across an electric field. It has units of $[C/m^2]$.

distribution in a vacuum, one needs to make a vector sum of all the forces due to all the small charges dq distributed over the region. This is written as

$$\vec{F}_0 = \frac{1}{4\pi\epsilon_0}\int \frac{q_0 dq}{r_2}\hat{r} = \frac{q_0}{4\pi\epsilon_0}\int \frac{dq}{r_2}\hat{r}$$

where $\hat{r}$ is a variable unit vector that points from each dq to the location of q_0.

4.5. Apparatus

The Coulomb balance by Pasco is shown in figures 4.1 and 4.2. Two conducting spheres are charged with an high-voltage DC power supply, and the force on one sphere due to the presences of the other charged sphere is measured using a torsion wire. The second sphere is mounted on a slide assembly allowing the distance between spheres to be changed. Owing the small magnitude of the obtainable electrostatic force, the apparatus is delicate and subject to interference from other electrostatic forces, as well as air currents. Only with careful use, calibration and the appropriate precautions will good results be obtained.

Figure 4.1.: Coulomb's Law apparatus

4.6. Method

In general, the apparatus is used by:

1. Charge the spheres with the same polarity of voltage to produce a repelling force.
2. Measure this force by turning the torsion knob to push the wire-suspended sphere back to its original position. Record the amount the knob is turned in degrees. Two forces are then balanced: the torsion force due to twisting the wire, and the electrostatic force due to electrical charges on the spheres.
3. The wire's torsion force is measured in a separate experiment which involves using small masses to obtain a constant in $\frac{newtons}{degrees}$. This constant is used to convert the degree reading from the torsion knob into a force.

Please read on for specific test instructions in order to start taking data.

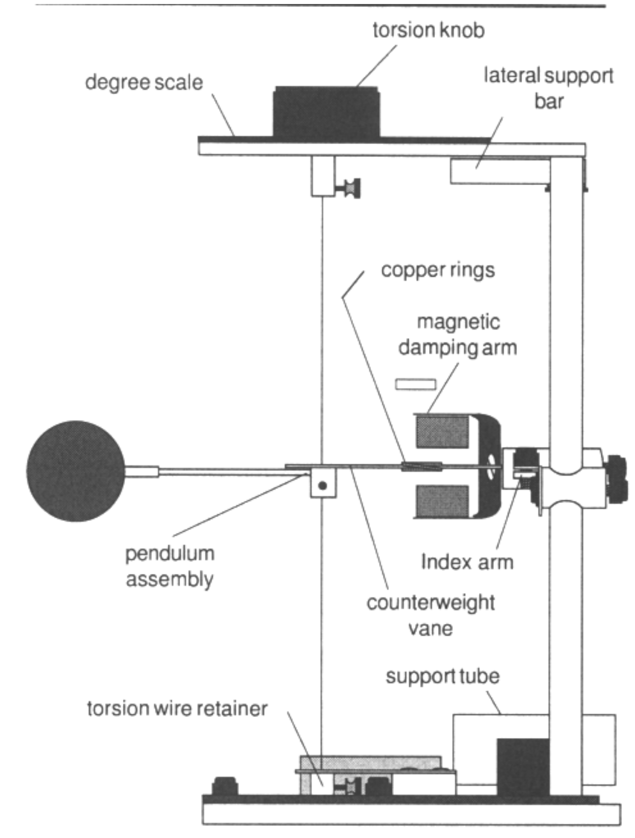

Figure 4.2.: Coulomb Law Apparatus parts.

Calibration

In order to detect only the electrostatic force, the instrument must be balanced to eliminate gravitational and the influence of other forces. These influences to the sphere position must be minimized or eliminated. In addition, the zero angle position for the two spheres must be correctly set.

4.6.1. Tips for accurate results

During use and calibration, air currents and external electrostatic forces can be a problem. The following general precautions and advice will assist the experimenter.

1. Drafts in the room are hopefully eliminated or minimized by redirecting ventilation airflow and by closing doors.

2. Have only one person near the apparatus at any time. Static charges on clothing (particularly synthetic materials) can have dramatic effects on the moving sphere's position. Cotton material is less likely to be statically charged.

3. The experimenter's movement near the apparatus should be minimized as walking by or waving a hand can create enough air movement to disturb a sphere's position.

4. Measurements should be made reasonably quickly as charge will dissipate into the surrounding air etc. (A test for what is a reasonable time will be performed after calibration.) Higher humidity increases charge loss to the air.

5. After charging the sphere, move the HV probe away from the spheres and turn down the HV supply voltage.
6. Walls and metal surfaces can have image charges induced on them from the sphere's charge. Keeping the apparatus away from walls is highly preferable.

4.7. Torsion Balance Setup

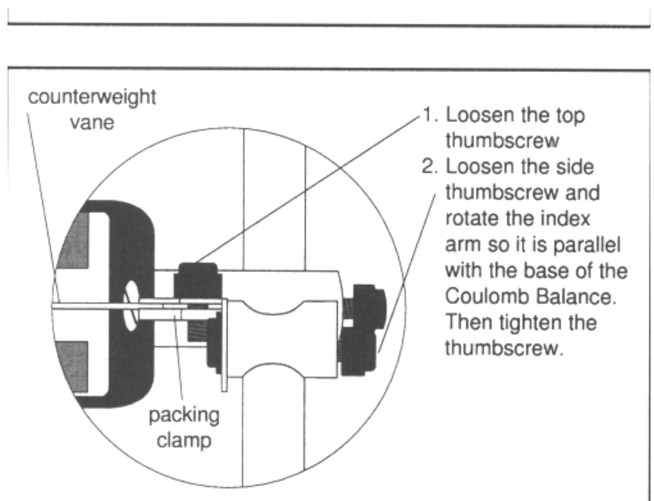

Figure 4.3.: Detail of packing clamp on the Coulomb apparatus.

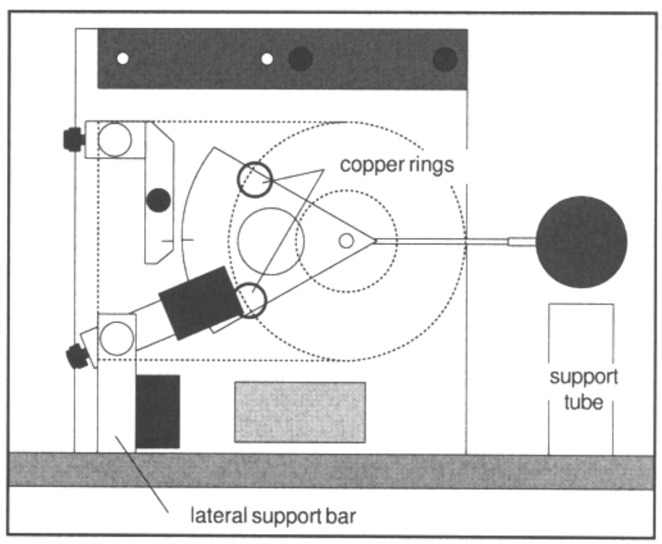

Figure 4.4.: Coulomb apparatus laying on its side.

1. The conductive spheres will already be attached to their support rods. Two copper rings will also be attached to the counterweight vane. The sliding track with its sphere and mount should not be attached to the torsion section until step 7. *Do not loosen the torsion wire retainer screw (at the bottom of the wire) which is keeping tension on the suspension wire.*

2. If present, release the packing clamp that holds the counterweight vane, as shown in figure 4.3. Release the vane from the clamp used for storage purposes. This clam forms the zero index for reading zero angle. The vane with its conducting sphere should be free to move now. Adjust the position of the copper rings so the pendulum assembly is level. The height of the magnetic damping arm should be checked so that the vane passes between the magnets.
3. Reposition the index arm so it is roughly parallel with the base of the torsion balance and at the same height as the vane.
4. Consult figure 4.2. Rotate the torsion dial until it reads zero. Note the counterweight vane has a black line on the end. This line should line up with the line on the index arm. Arrange for this condition as shown by the lab instructor. Rotation of the bottom torsion wire retainer will be necessary. Do not loosen the screw, but use it as a "handle" to rotate the wire/vane assembly and align the two marks.
5. Extend the lateral support bar and tip the apparatus on its side. The counterweight vane's line should still line up with the line on the clamp. Slide the copper ring(s) in the correct direction until this condition is achieved.
6. Stand the apparatus up again and repeat steps 4 and 5 again to double check balance and zero. This should be a reasonably quick process. If not, check section 4.6.1 for assistance.
7. Stand the apparatus upright and attach the sliding sphere with its base to the torsion balance section. Consult figure 4.5.
8. The spheres should be aligned horizontally and vertically. A small difference is tolerable. Set the sliding sphere to the 3.8cm position. The sphere should then just or barely touch as this distance is equal to the diameter of a sphere. If there is alignment problems, consult your lab instructor.

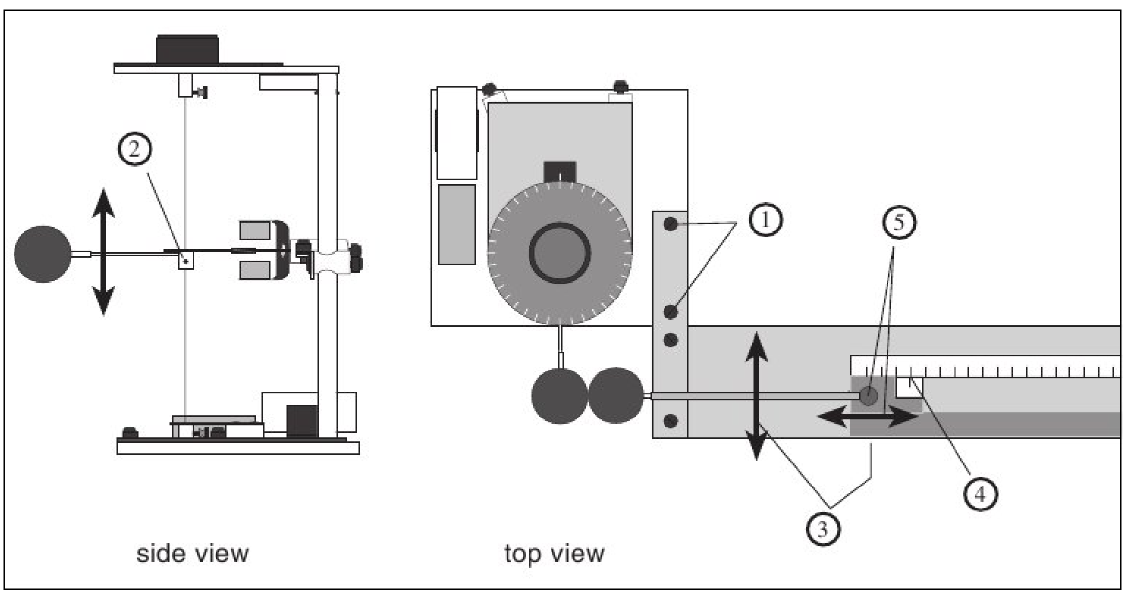

Figure 4.5.: Adjustments to align the spheres of the Coulomb apparatus.

The apparatus should now be ready to use. If the sphere's position keeps changing, then check that drafts are not causing this. If the sphere is not still, the apparatus is not going to produce good results. You will gradually train yourself on taking data which is reliable.

4.8. EXPERIMENTS

There are two experiments involving charging described below. A third procedure must be completed to convert angles into forces. Complete all for the lab report.

4.8.1. Force as a Function of Distance

As a test of the apparatus, first determine if repeatable results are possible.

1. Set the separation "r" of the centers of the spheres to $4.8cm$. Be careful when setting this. The distance read on the slider is equal to the distances between the center of each sphere. A separation of $4.8cm$ means the the near sides of each sphere will be $1.0cm$ apart.

2. Charge both spheres to $6000V_{DC}$. If the sphere being charged then contacts the other sphere, recharge it. The contact will result in sharing of charge. When the second sphere or object is not charged, then the sphere just charged does not have a full charge after contact.

3. Rotate the torsion dial far enough to get the vane's line and the index arm's line to coincide. Record the angle rotated by reading the dial. This measures the repulsive force between the charged spheres.

4. Discharge both spheres and set the dial back to $0°$.

5. Repeat the above steps. If you get reasonably repeatable results, then you can proceed. If not, insure that you are following all the precautions listed in subsection 4.6.1.

Equation 4.2 indicates the force of one charge on another charge or charges has a $\frac{1}{r^2}$ dependence. The goal is to verify this dependence. Collect data as follows.

1. Your first data point can be one of the trials from the above test.

2. Increase the separation r to $5.8cm$.

3. Charge both spheres to $6000V_{DC}$. Estimate a reasonable error for the voltage values.

4. Measure the angle required to force the two lines (vane and index lines) to coincide. Again make a judgement on what the error is on this angle.

5. Increase in steps of about $1.0cm$ to $2.0cm$ and collect data for another seven separations. Record a value for the error in separation.

4.8.2. Torsion constant k_T

So far the angle θ has been assumed to be proportional to the electrostatic force. This can be verified by using small masses placed on the sphere's center line to act as weight, thus applying a known force to twist the wire. In this way the relationship between the angle θ and force can be found, and values of θ and more importantly, $\theta_{corrected}$ can be converted to force values in newtons.

1. Carefully turn the Torsion Balance on its side, supporting it with the lateral support bar, as shown in figure 5. Place the support tube under the sphere, as shown. It will prevent it from swinging downward too far which slows the procedure down.

2. Zero the torsion balance by rotating the torsion dial until the index lines are aligned. Record the angle of the degree plate as weight is added to the conducting sphere.

3. Carefully place the 20 mg mass on the center line of the conductive sphere. Turn the degree knob as required to bring the lateral support bar index lines back into alignment. Read the torsion angle on the degree scale. Record the angle.

4. Repeat step 3 using the other available milligram masses. Each time record the mass and the torsion angle needed to bring the index lines into alignment.

4.8.3. Force as a function of Charge

For this test, the spheres will be at different voltages. It is now important that the spheres do not touch after being charges as that will put them at some equal value of charge which will not be known.

1. With the sphere separation r held at a constant value, charge the sphere to different values of using different voltages and measure the resulting force. Keep the charge on one sphere constant and charge the other sphere to different potentials in steps of $500V_{DC}$.

2. Start by charging both spheres to a voltage near the maximum available. Use a small separation of about 4 to 5 centimeters. Measure θ as before.

4.9. Analysis

4.9.1. Correction Factor B

The inverse square law for the force's (and hence θ's) dependence on separation r will fail for small r for this apparatus. Equation 4.2 is for point charges, but the apparatus is using spheres whose size cannot be considered trivial for small r. Additionally the charge on a sphere will not distribute evenly over the sphere owing to the presence of the second charged sphere. Instead, the charge will redistribute itself on both conducting spheres to minimize electrostatic energy. The force will actually be less than expected as the two spheres are being charged with the same polarity of charge which causes repulsion. Thus the "center of charge" for each sphere is further away than the physical centers of the spheres.

1. The value of θ can be corrected using a factor of B where

$$B = \frac{1}{1 - \frac{4a^3}{r^3}}$$

   where $a = 1.9cm$ is the radius of the spheres and r is the separation between the spheres.

2. Create column for the B and σ_B values in your report's table.

3. Multiply all values of θ by B and call this value $\theta_{corrected}$. Calculate error for these corrected angles $\sigma_{\theta,corrected}$.

4.9.2. Calculation of Torsion constant k_T

These values of θ do not require correcting.

1. Calculate the weight for each set of masses and the resulting force F_w that you used.

2. Plot a graph of F_w as a function of θ and determine the slope with error. Call this value of the slope $k_T \pm \sigma_{k_T}$. Does the graph show a linear relationship between the angle and the force?

3. Use k_T to convert your corrected angles into a force in newtons for the following analysis sections.

4.9.3. Force as a Function of Distance

If $\theta = cr^n$ where c and n are unknown constants, then $\log\theta = \log c + n\log r$. The slope of a plot of logθ as a function of log$r$ will be a straight line of slope n and the y-intercept of log$c$. Thus if the plot gives a straight line, the function has been verified.

1. Apply the correction factor to the θ values as outlined in section 4.9.1. For small r, the data will not follow the inverse square law, making it necessary to apply this correction factor.
2. Plot a graph oflog$_{10}(\theta_{corrected})$ as a function of $\log_{10} r$. State your values for c and n with error.
3. Compare n to the expected value.
4. What variables and constants does the value of c provide a value for?

4.9.4. Force as a function of Charge

Note that calculating the charge q is not possible without a value for the capacitance of the spheres. However, plotting force as a function of voltage (on the sphere which was charged to different voltages) will achieve the same goal as $q = CV$ where C is a constant representing the capacitance of the sphere.

1. Apply the correction factor to the values of θ. The torsion constant k_T must be used on the angles.
2. Plot a graph of force in newtons (use $\theta_{corrected}$) as a function of charge to determine the relation between these quantities.
3. State the relationship between the plotted variables. Quote the equation from the best fit line with error.
4. Does the data show that force is proportional to charge?