7. Variation of Resistance with Temperature

7.1. Theory

7.1.1. Copper

The electrical resistance of most metals increases nearly linearly with increasing temperature. The resistivity of copper is shown as a function of temperature over a large temperature range in Figure 7.1.

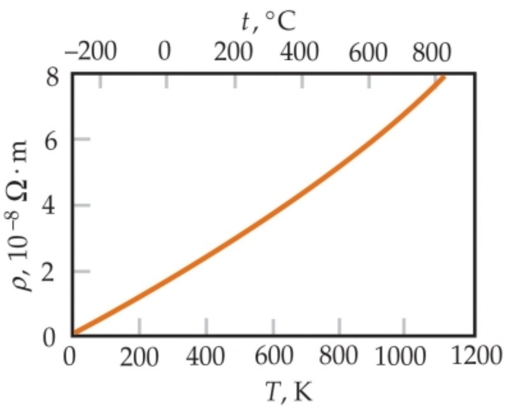

Figure 7.1.: Electrical resistivity of copper as a function of temperature (Fig. 25-5 from *Physics for Scientists and Engineers, 5th Edition*, Tipler and Mosca)

Over a smaller temperature range, the resistance of most metals can be closely approximated by the linear relation:

$$R = R_{ref} + \alpha R_{ref}(T - T_{ref}),$$

where R_{ref} is the resistance at some reference temperature T_{ref}, T is the temperature in degrees Celsius and α is the *temperature coefficient of resistivity* which can be defined as the ratio shown in equation 7.1.

$$\alpha = \frac{R - R_{ref}}{R_{ref}\,(T - T_{ref})} \tag{7.1}$$

The reference temperature is usually $20°C$, but $0°C$ is also used. One can also use $T_{ref} = 0C = T_0$ and $R_{ref} = R_0$ and therefore equation 7.2 will apply.

$$R = R_0 + \alpha R_0 T \tag{7.2}$$

For this lab, we will use the lowest temperature attained as the reference temperature. Thus R_{ref} is R at that lowest temperature.

7.1.2. Thermistor

Thermistors are thermally sensitive resistors usually made of a semi-conducting material. They have resistance values in the range between those of conductors and insulators. They also have the property that their resistance *decreases* very rapidly with increasing temperature. This occurs because the charge carriers gain enough thermal energy to allow them to cross the energy gap between the valence and conduction bands when the temperature is increased. The result is that there are more mobile charges to take part in the conduction process. Thermistors are *extremely* sensitive to even small changes in temperature and have good stability. For these reasons, thermistors are very useful for temperature measurements, temperature control, voltage regulation, safety and warning circuits and many other applications. The resistance vs temperature curve for a specific thermistor is shown in Figure 7.2.

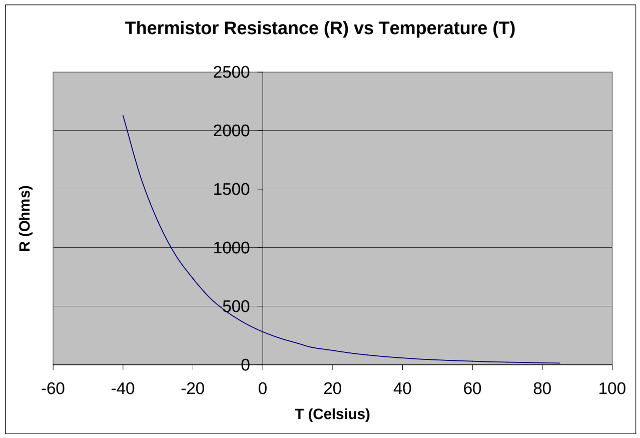

Figure 7.2.: Resistance vs Temperature curve for Sensor Scientific Inc. Standard Epoxy Coated Thermistor (W101A10) (*www.sensorsci.com/RVT_SCT.htm*)

The relationship between resistance and temperature is inherently non-linear and is given by the equation 7.3.

$$R = R_{ref}\, e^{\beta\left(1/T - 1/T_{ref}\right)} \tag{7.3}$$

where R_{ref} is the resistance at the reference temperature T_{ref} and both temperatures are absolute temperatures measured in Kelvins. The parameter β is a characteristic property of a particular semi-conducting material which determines the variance of its resistance with temperature over a limited temperature range. Equation 7.3 can be rewritten in the form:

$$\ln\left(\frac{R}{R_{ref}}\right) = \beta\left(\frac{1}{T} - \frac{1}{T_{ref}}\right).$$

7.1.3. Apparatus

- Thermistor assembly
- Copper coil assembly
- Heating unit with water bath
- Extech Precision MilliOhm Meter
- Thermometer
- glycerin

You will determine the value of the temperature coefficient for copperα, and the value of β which is a characteristic of the thermistor material. There are four principal parts supplied for the experiment which

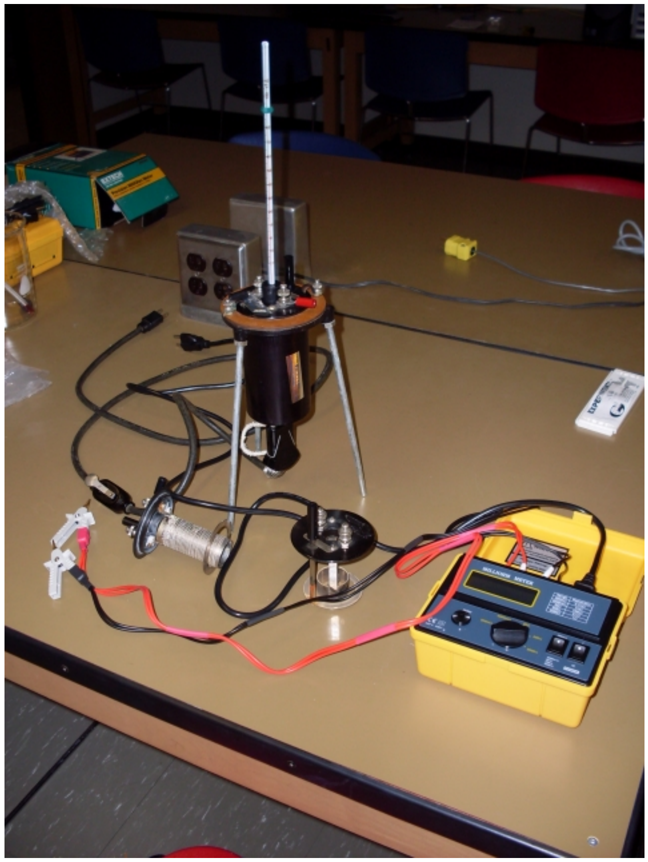

Figure 7.3.: Apparatus for temperature dependence lab.

are shown in Figure 7.3. These are a thermistor assembly, a copper wire assembly, an electrically heated water bath and a milliohmmeter. The thermistor assembly consists of a disc type thermistor attached to binding posts on a Bakelite disc. This disc serves as the water bath cover and includes a stirrer and a rubber stopper. The copper wire assembly is essentially the same, except it has a copper wire coil wound on a brass cylinder instead of a thermistor unit. Both are specially coated for protection against moisture.

The water bath is a metal vessel with a permanently mounted, $250\,W$ immersion heater. It will be filled with water in which you will immerse either the copper or thermistor unit and the temperature will be varied by means of the heater. The vessel is supported by a tripod to which the vessel and the element under test should be clamped. A Kelvin clip (four-wire) milliohmmeter is used to measure the test object's resistance. The temperature is monitored by a standard glass thermometer.

The milliohmmeter (see Figure 7.4) is specially designed for accurately measuring resistances in the $m\Omega$ to Ω range. The resistance is measured by passing a small test current (see Table 1) from the HI to LO clip lead (see Figure 7.5) which will pass through the device under test. Once the test current is applied, the voltage drop across clip leads Rx1 and Rx2 is measured. The resistance is then the ratio of the voltage drop to the test current and will appear on the LCD display.

7.2. EXPERIMENT

7.2.1. Safety Warning

The experiment will result in the water being at or above $80°C$. This is hot enough to cause serious burns to a person's skin. Do not accidentally bump the apparatus and tip it over during the course of the experiment.

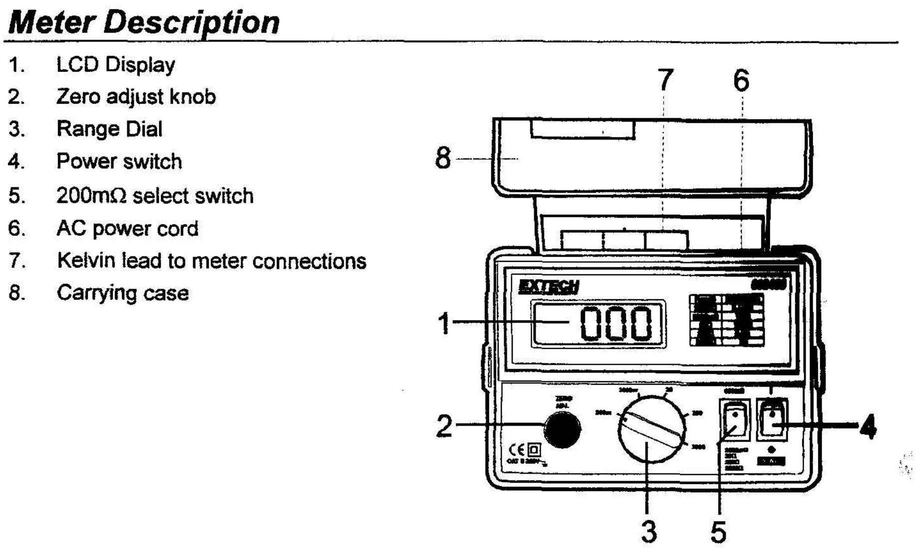

Figure 7.4.: Extech Precision MilliOhm Meter

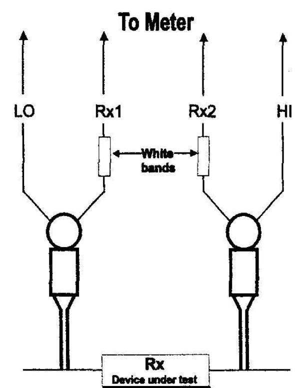

Figure 7.5.: Lead attachment for Extech MilliOhm Meter

Precautions

1. Be sure that the thermometer is completely immersed in the water bath. Have it situated as close to the thermistor as possible or in the center of the copper coil.
2. Let the temperature reach a maximum (ie stop changing) before taking resistance measurements.
3. Stir the water well, especially just before reading temperature and resistance.

7.2.2. Equipment Set-up

1. Connect meter leads as shown in Figure 7.5, plug in the milliohmmeter and turn it on.
2. Rotate the range select switch to the $200m\Omega$ range to prepare for zeroing and ensure that $200\,m\Omega$ is selected on the switch adjacent to power switch.
3. Zero the milliohmmeter by shorting the Kelvin clips as shown in figure 7.6 and rotate the Zero Adj. knob until the meter indicates zero units.

Specifications

General Specifications

Circuit	Custom one-chip LSI microprocessor circuit
Display	0.7" (18 mm) LCD with 2000 display counts
Measurement terminals	4-Terminal Kelvin type
Measurement ranges	Five (5) ranges (200m, 2, 20, 200, 2000Ω ohms)
Zero Adjust	±50 count adjustment
Sampling Time	Approximately 0.4 seconds
Over input indication	Indication of "1 - - -"
Operating Temperature	32 °F to 122 °F (0 °C to 50 °C)
Operating Humidity	80% Relative Humidity max.
Power Supply	110V (380460) or 220V (380462) ±15%, 50/60Hz
Power Consumption	Less than 2 VA
Weight	1.32 lbs (600g)
Dimensions	6.3 x 4.72 x 3.35" (160 x 120 x 85 mm) with cover
Accessories	Power Cable and 4-wire Kelvin clip leads

Range Specifications

Range	Resolution	Test Current	Accuracy	Open Circuit Voltage
200 mΩ	0.1mΩ	100mA	± 0.75% + 4 digits	3.8V
2 Ω	1mΩ	10mA	± 0.75% + 2 digits	3.4V
20 Ω	10mΩ	10mA	± 0.75% + 2 digits	3.4V
200 Ω	0.1 Ω	1mA	± 0.75% + 2 digits	3.2V
2000 Ω	1 Ω	1mA	± 0.75% + 2 digits	3.2V

Note: Specifications tested using RF Field Strength <3V/m and frequency <30MHz

Table 7.1.: Extech MilliOhm Meter specifications

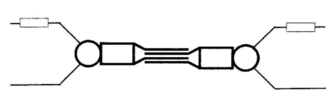

Figure 7.6.: Shorting the Kelvin clips

4. To measure resistances outside this range, switch the $200\,m\Omega$ select switch to the other position and adjust the range select dial appropriately.

5. Set up the tripod support and fill the vessel with cold water to within an inch of the top.

6. Support the vessel on the fiber ring on the tripod.

7. Using the one-hole rubber stopper, mount the thermometer through the Bakelite cover of one of the units (either the thermistor or copper wire).

   - Note: Glycerin is required as a lubricant to insert and usually to remove the thermometer.

8. Check that the thermistor or wire is completely immersed.

9. Secure the apparatus by tightening the two clamps.

10. Attach spade clips to the binding posts of the heating unit to provide some exposed metal for the Kelvin clips.

11. Attach the AC power cord to the bottom prongs of the heating unit and plug it into the AC 115V receptacle.
    - Note: The cord switch is on when the kept depressed and off otherwise. This avoids inadvertent burn-out of the heater.
12. Attach the Kelvin clips from the milliohmeter as securely as possible to the spade clips.

The apparatus should now be ready to use.

7.2.3. Procedure

1. Measure the unit's resistance on the milliohmmeter and the temperature to within $\frac{1}{2}$ a degree.
2. Construct a data table for the copper wire showing temperature T in degrees celsius and resistance R in ohms. Use the heater to raise the temperature by about six to eight degrees celsius. (Note: Turn off the heater one or two degrees before the desired temperature is reached.)
    - Stirring the water will ensure that the sensor and thermometer are both at the same temperature.
3. After turning the heat off, wait until the maximum temperature has been reached before taking readings of the temperature and resistance.
    - Note: When taking measurements with the thermistor unit, it is especially important that temperature and resistance measurements are taken virtually simultaneously. This is not as crucial when working with copper (because of its lower sensitivity), but ideally all temperature measurements should be taken this way.
4. Continue taking measurements at about six to eight degree temperature intervals until a temperature of about $80°C$ is reached.
5. Repeat the procedure for the other assembly.

7.3. Analysis

7.3.1. Copper

Convert all temperature readings to kelvin and plot a graph[1] of resistance *vs* temperature in kelvin using the data for the copper wire assembly. Determine the temperature coefficient of resistance α for copper and compare to the known value. Is your graph consistent with the theoretical relationship?

7.3.2. Thermistor

Construct a data table for the thermistor including raw data and all calculated data for graphing, *ie.* temperature in Celsius, temperature in Kelvin, inverse temperature $[K^{-1}]$, $\left(\frac{1}{T} - \frac{1}{T_{ref}}\right)$, resistance, ratio of resistance to reference resistance and $\ln\left(\frac{R}{R_{ref}}\right)$. Plot a graph of $\ln\left(\frac{R}{R_{ref}}\right)$ *vs* $\left(\frac{1}{T} - \frac{1}{T_{ref}}\right)$ for the thermistor. Determine the value of β for the thermistor over this temperature range. The manufacturers specifications for the thermistor are $\beta = (3530 \pm 80)\,K$. Is your thermistor graph consistent with the theoretical relationship?

[1] Should you plot R as a function of T or $R - R_ref$ as a function of $T - T_{ref}$? Consult the theory to decide.

7.4. Questions

Answer the following question in a separate section of your report.

1. What is the advantage of the kelvin clip meter leads for the milliohmmeter as compared to the common meter leads on a multimeter.
2. What effect will Joule heating from the test current flowing through the element (copper or thermistor) have?
3. If the copper wire is partially submerged in the water, what effect would this have on your results?
4. Look up the temperature coefficient of resistance for copper for reference. Why is there a range of values?