14. $\frac{1}{r^2}$ Law and Malus' Law

14.1. Object

- To demonstrate that the Illuminance from a point source obeys the $\frac{1}{r^2}$ law.
- To determine the relationship between transmittance of light and angle between polarizer-analyzer pair.

14.2. Point Source of Light

This experiment tests the behavior of a light source as the distance from that source is increased. The Pasco light sensor's output is a value of illuminance in lux. It is important to understand terms used in measuring light sources. The word brightness should not be used for quantitative work as it is subjective.

14.3. Theory and Definitions

In general terms, flux Φ is the number of particles flowing per unit area. In other words, flux represents the quantity which passes through a surface area. The concept of flux is applied to different areas of physics: luminous flux, electric flux and magnetic flux are quantities frequently used. It can even be used to quantify the amount of water that flows through a cross-section of a river per second. Figure 14.1[1] shows some variables that affect the value of flux.

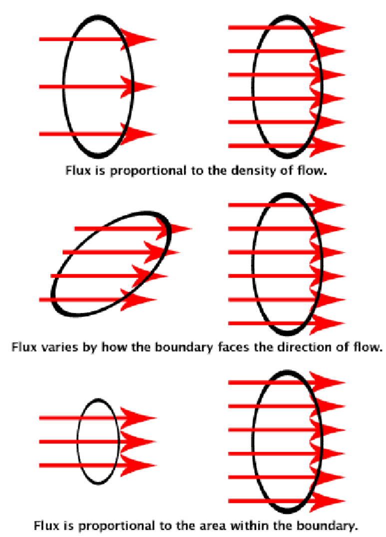

Figure 14.1.: Visualization of factors that affect flux.

As the distance increases from a light source, the source appears less bright. To simplify analysis, a source of light can be approximated as a uniform point source if its dimensions are negligible compared to other length scales present in the situation, and it has the characteristic of emitting light uniformly in all directions.

[1]Source: http://en.wikipedia.org/wiki/File:Flux_diagram.png

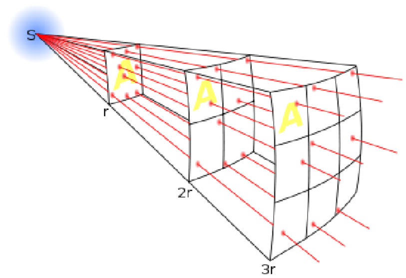

Figure 14.2.: Expanding surface area of a sphere with increasing radius.

Figure 14.2[2] shows a point source and how the density of lines through a solid angle remain constant. As the radius increases by a factor of two, the area that the lines pass through becomes four times larger. If the radius is tripled then the area becomes nine times larger. This results from the fact that the the relationship between surface area S of a sphere and its radius r is $S_{sphere} = 4\pi r^2$. Note how the number of flux lines penetrating area A drops with increasing radius.

A steradian (sr) is the unit of solid angle subtended by an area on the surface of a sphere equal the square of the radius of the sphere. One steradian can be visualized as a conic section with a solid angle of 65.56°. The steradian is dimensionless like the radian and is analogous to the radian definition $\theta = \frac{l}{r}$. Capital omega (Ω) is used to denote a solid angle. The definition of solid angle is shown in equation 14.1.

$$\Omega = \frac{S}{r^2} \tag{14.1}$$

There is no demand for the area S to be perfectly circular. It is the magnitude of S that defines the solid angle Ω.

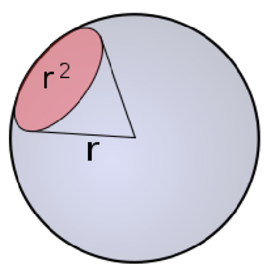

Figure 14.3.: Defining the steradian. The circular area $S = r^2$.

In terms of solid angle, a complete sphere has 4π steradians. It follows that a sphere of radius 1 meter has a surface area of 4π square meters. Thus a solid angle of $1\,sr$ encompasses an area of $1\,m^2$ when the radius is $1\,m$.

Luminous Intensity (symbol I, measured in the SI units of candelas[3]). Luminous intensity I_v is a measure of the light-emitting ability of a light source, either generally or in a particular direction. A candela (Cd) is defined as being equal to the luminous intensity in the perpendicular direction of the black-body radiation for an area of $\frac{1}{600{,}000}m^2$ at the temperature of freezing platinum under a pressure of 101325 pascals. Conversion between candela and lumens per steradian is done using the following relation.

$$1\,candela = 1\frac{lumen}{steradian}$$

Since the total solid angle subtended at a point is 4π steradians, a source whose intensity in all directions is $1\frac{lumens}{steradian}$, emits a total of 4π lumens.

[2]Source: http://en.wikipedia.org/wiki/File:Inverse_square_law.svg

[3]This unit replaces the old candle standard unit which was the first photometric standard based on a candle constructed in specific way.

The *Radiant Flux* (symbol Φ_e, units of watts) is the term used to quantify the total power emitted, received or passing in the form of electromagnetic radiation. For a uniform point source whose output radiant flux is constant over time, concentric spheres surrounding that source would all receive the same total amount of flux. However, each unit area of the spheres would receive decreasing amounts of flux as the radius increases, following the $\frac{1}{r^2}$ law: $\Phi_{e\,at\,r} = \frac{\Phi_{e\,total}}{4\pi r^2}$.

The *Radiant Intensity* (symbol I_e, measured in $\frac{W}{sr}$) is the radiant flux per unit solid angle emitted by a source.

Luminance (photometric brightness, symbol L, units of $\frac{candela}{m^2}$) is defined as the luminous intensity I_v of any surface area S in a given direction per unit projected area of the surface, viewed from that direction and $L = \frac{dI_v}{dS\cos\theta}$. θ is the angle between the line of sight and the normal to the surface area. In this case, the surface is emitting light.

Luminous Flux (symbol Φ_v, units of lumens) is a measure of the rate of flow of light, i.e. the radiant flux in the wavelength range 380-760 nanometers, corrected for the human eye's sensitivity dependence on wavelength. It is measured by reference to emission from a standard source. The lumen is the SI unit of luminous flux and equals the flux emitted by a uniform point source of 1 candela in a solid angle of one steradian.

Illuminance (symbol E, measured in lux): When light (luminous flux) strikes a surface, we say that the surface is illuminated. Illiminance is the luminous flux per unit area:

$$E = \frac{d}{dS}\Phi_v \tag{14.2}$$

One lux is equal to the illumination produced by a luminous flux of one lumen distributed uniformly over the area of 1 square meter: $1\,lux = 1\frac{lumen}{m^2}$. In this case, the surface is being lit by a light source. If the illuminance is the same at all points of a surface of finite area S, and if Φ is the total luminous flux incident on the surface, equation 14.2 becomes

$$E = \frac{\Phi}{S}$$

Since S scales linearly with r^2 as shown in equation 14.1, the illuminance will have a $\frac{1}{r^2}$ dependence. Typical values of illuminance are shown in Table 14.1.

Type of Illumination	Typical Illuminance in lux
Sunlight plus skylight (maximum)	100,000
Sunlight plus skylight (dull day)	1,000
Interiors - daylight	200
Interiors - artificial light	100
Full moonlight	0.2

Table 14.1.: Typical values of Illiminance

14.4. Pasco Light Sensor and Light Source

The Pasco Light Sensor PS-2106A measures illuminance E in lux. It has three ranges, 0 - 2.6 lux, 0 - 260 lux and 0 - 26000 lux and samples at a default rate of 5Hz. Figure 14.4 shows its directivity. The Pasco Light Source OS-8470 uses a 12V, 10W G4 halogen bulb. Its filament is approximately 3mm across.

14.4.1. Equipment

60cm or 1m Pasco Optics Bench, two Pasco basic optics accessory holders, two polarizers for accessory holders, Pasco Basic Optics light source OS-8470, GLX Data logger, Passport Light Sensor PS-2106A with extension cable and ~5cm mounting post, desk lamp, masking tape, small foam or other object to block extraneous light, two ~30cm aluminum rods, three 90° clamps, one heavy-base lab stand.

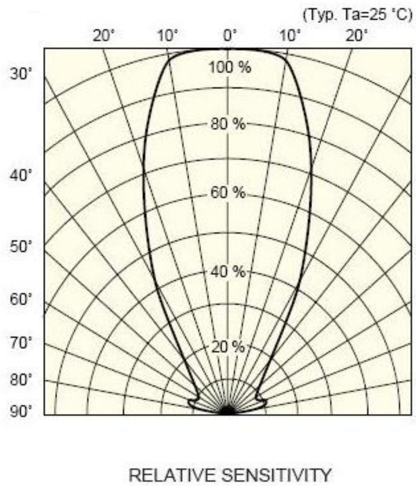

Figure 14.4.: Directivity of the PS-2106A Light Sensor

14.5. Experiment: $\frac{1}{r^2}$ Law Test of Illuminance

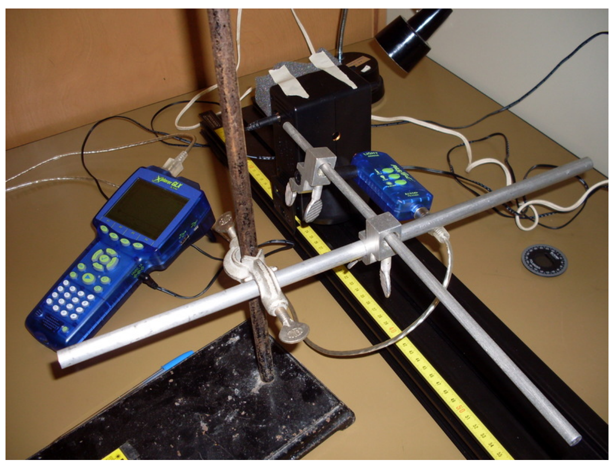

Figure 14.5.: Equipment setup for $1/r^2$ test

14.5.1. Procedure

Setup the equipment as shown in Figure 14.5. The GLX has a back-light for the display if you need it. It is controlled in the Settings icon of the Home screen.

1. Depending on choice of light source there may be light emitted out the back where a grid-like pattern is produced. Cover this over to prevent it from contributing light to the sensor.

2. Align the sensor with the light source by using the GLX in Digits mode and maximizing the reading. Problems of readings remaining constant usually mean the sensor is set on the wrong scale. Three buttons on the sensor control the range.

3. With the GLX in Digits mode, bring the sensor right up to the source and take the first reading. Record the illuminance for every cm increase of separation out to 18 cm.

14.5.2. Analysis

Plot a graph of illuminance as a function of $\frac{1}{r^2}$ where r is in meters. Grace or SciDavis can be used. Fit the data with the line of best fit, and quote the errors. Carefully examine the data for points that may not fit the line. Explain any discrepancies between your data and the $\frac{1}{r^2}$ theory.

14.6. Linearly Polarized Light and Malus' Law

14.6.1. Background

A polarizer only allows light which is vibrating in a particular plane to pass through it. This plane forms the "axis" of polarization. The transmitted light will be polarized, meaning the the electric field vector will have one direction of vibration.

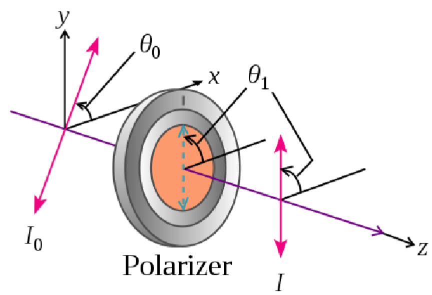

Figure 14.6.: Incident polarized light and polarizer filter. Here $\theta_i = \theta_1 - \theta_0$

Etienne-Louis Malus formulated a mathematical description of the transmitted intensity I for polarized light incident on a polarizer. If the incident polarized light has intensity I_0, one can compute I using equation 14.3,

$$I = I_0 \cos^2 \theta_i \tag{14.3}$$

where θ_i is the angle between the incident light's polarization direction and the axis of the polarizer. Figure[4] 14.6 shows the configuration of the variables.

If unpolarized light is incident upon an "ideal" polarizer, only half of the light will be transmitted through the polarizer. Since there are no "ideal" polarizers, less than half the light will be transmitted. Since the average value of $\cos^2 \theta$ is $\frac{1}{2}$ the statement above holds.

For this experiment, two polarizing filters will be used. The first filter will transmit linearly polarized light to a second filter. This second filter's orientation will be changed. The variable θ will be used to denote the angle between the two polarizers axes and takes the place of θ_i in Equation 14.3.

14.6.2. Equipment

60cm or 1m Pasco Optics Bench, Pasco Basic Optics light source OS-8470, GLX Data logger, Pasport Light Sensor PS-2106A with extension cable, two polarizers mounted on accessory holders for bench, lab stand, 90° clamp, finger clamp, desk lamp, lab stand, two ~30cm aluminum rods, two 90° clamps.

14.6.3. Procedure

Set up the equipment as shown in Figure 14.7. (The extension cable is not shown as it was unavailable at the time and thus the lab stand and clamps were not used.) The light sensor can be held in position with the finger clamp mounted on the lab stand.

1. Position the light sensor for maximum reading after choosing the appropriate range of the three available on the light sensor.

[4]Diagram from: http://en.wikipedia.org/wiki/File:Malus_law.svg

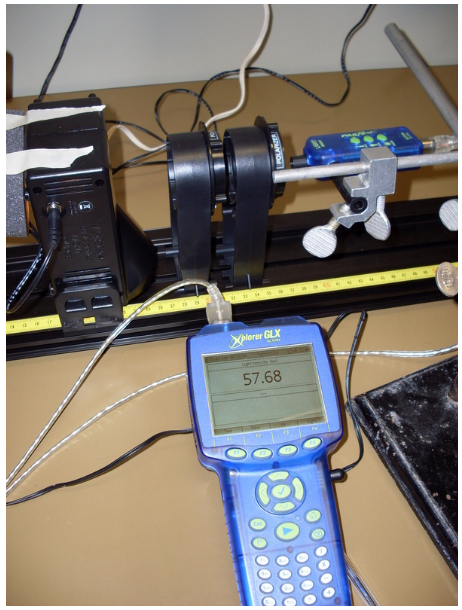

Figure 14.7.: GLX and equipment for polarized light experiment.

2. Rotate the two polarizers to achieve maximum transmission of light to the light sensor. The two polarizers' axis are now aligned, so the angle between them is zero. The equipment should not be moved from now on, with the exception of the analyzer's angle.

3. Take readings every 10° of rotation of the analyzer, for a total of 400° rotation.

14.6.4. Analysis

1. Use xmgrace or scidavis to fit a $\cos^2\theta$ curve of form $y = a_1 + a_2[\cos(a_3 + a_4\theta)]^2$ to the data. This can be made in either scidavis or xmgrace. See Figure 14.8 for suggestions for xmgrace and Figure 14.9 for suggestions for using scidavis. These graphs use data from the GLX.

2. Explain any discrepancies in the graph.

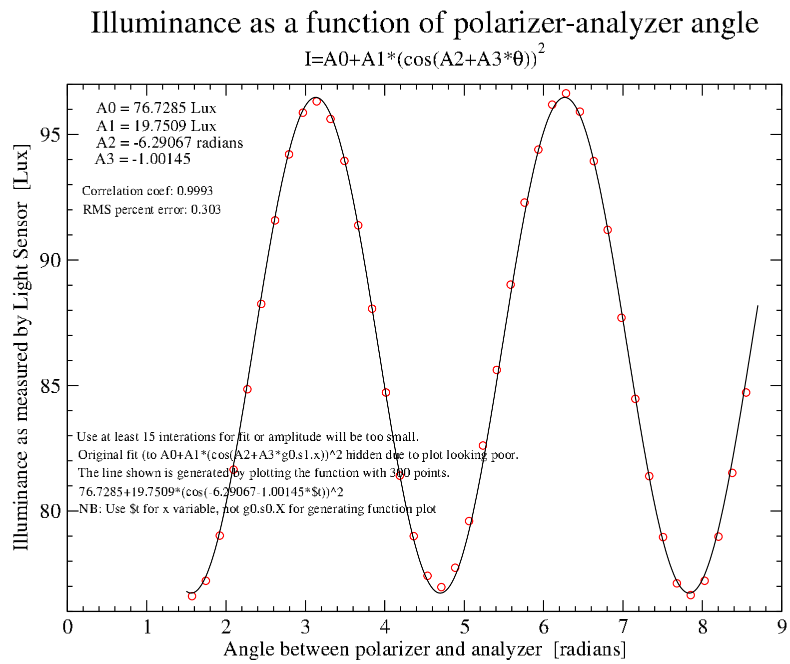

Figure 14.8.: Graph of polarizer experiment data with useful tips for plotting in xmgrace.

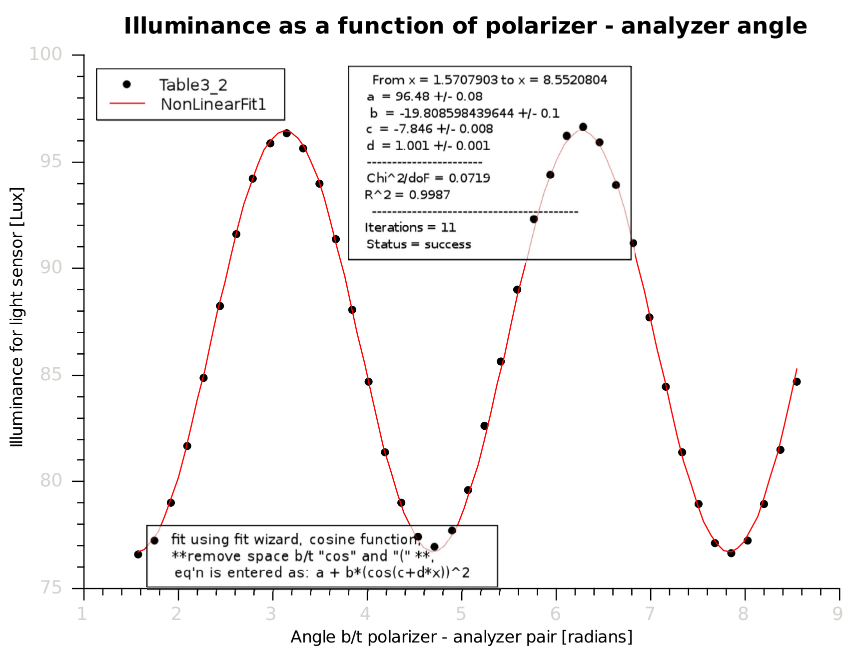

Figure 14.9.: Graph of polarizer experiment data with useful tips for plotting in scidavis.