1 General Lens Equation

1.1 Equipment

water, beaker, small weight (dollar coin, etc), eye-dropper (optional), from Basic Optics Kit: Light Source (rays), hollow lens, clear plastic box (items removed), white plastic sheet

1.2 Purpose

To explore the properties of a lens by changing the index of refraction of the lens and the surrounding media.

1.3 Theory

A conventional lens is made of material whose index of refraction is higher that the air surrounding it. Examples of this include magnifiers and corrective glasses which may be made of plastic or glass. Most glass materials have an index of refraction between roughly 1.4 and 1.9, while air is about 1.00029. However a lens can also made of material whose index is less than the surrounding medium. In this lab a "hollow" lens of air will be partially or fully surrounded by water whose index of refraction is 1.333 at $589nm$. This will be compared to a lens of water partially or fully surrounded by air.

Figure 1.1[1] shows a lens of material n' situated between two media of different refractive indices n and n''.

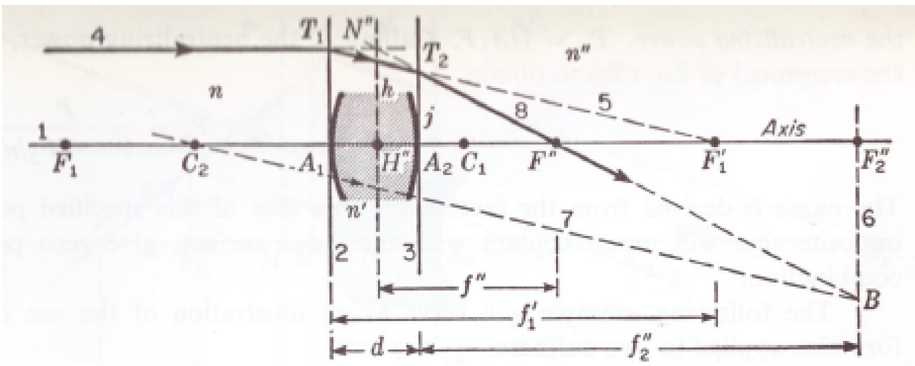

Figure 1.1: general lens parameters.

C_1 and C_2 are the centers of the spherical surfaces of the lens; The respective radii of curvature begin at these centers and reach across to the far surface which is their matching lens surface.

F'' is the focal point of the lens (both refractive surfaces are involved)

F_1' is the focal point for rays striking the first surface (left) and continuing forever in glass thereafter. Thus these ray never see the second surface.

F_2'' is the focal point for parallel rays striking the second boundary (right). Thus these rays don't involve refraction at the left (first) surface.

The Gaussian formula[2] for a thick lens is used as a starting point and relates the focal lengths to the indexes of refraction and the thickness of the lens d. For the first surface:

[1] p85, Jenkins & White, Fundamentals of Optics, Fourth Ed.
[2] p84, Jenkins & White, Fundamentals of Optics, 4th Ed.

$$\frac{n}{f} = \frac{n'}{f'_1} + \frac{n''}{f''_2} - \frac{dn''}{f'_1 f''_2} \tag{1.1}$$

and similarly for the second surface,

$$\frac{n''}{f''} = \frac{n'}{f'_1} + \frac{n''}{f''_2} - \frac{dn''}{f'_1 f''_2} \tag{1.2}$$

Treating d as small compared to the product of $f'f''$ allows us to drop the last term in equation 1.1. Using the fact that for each single surface of the lens we know that $\frac{n'}{f'_1} = \frac{n'-n}{R_1}$ and $\frac{n''}{f''_2} = \frac{n''-n'}{R_2}$, we have for the first focal point[3]:

$$\frac{n}{f} = \frac{n'-n}{R_1} + \frac{n''-n'}{R_2} \tag{1.3}$$

When the media on either side of the lens are different, the two focal lengths are different and have a ratio corresponding to their refractive indices[4]. This means

$$\frac{n}{f} = \frac{n''}{f''} \tag{1.4}$$

Note that for $n \neq n''$ then it must be that $f \neq f''$. However the ratios are equal. Thus the f-points of a thick lens are not symmetric.

Using the right-hand sides of equations 1.3 and 1.4:

$$\frac{n''}{f''} = \frac{n'-n}{R_1} + \frac{n''-n'}{R_2} \tag{1.5}$$

With air on both sides of the lens ($n = n" = 1$ and now f'' becomes simply f'), we have the familiar lens-maker's formula:

$$\frac{1}{f} = \frac{1}{f'} = (n'-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \tag{1.6}$$

Returning to the case of a thick lens, Equation 1.6 becomes

$$\frac{1}{f} = \frac{1}{f'} = (n'-1)\left(\frac{1}{R_1} - \frac{1}{R_2} + \left(\frac{(n'-1)d}{n'R_1R_2}\right)\right) \tag{1.7}$$

The equation relating the object distance s and the 1.7 image distance s' is[5]:

$$\frac{f}{s} + \frac{f'}{s'} = 1 \tag{1.8}$$

1.4 Questions on theory

1. Use equations 1.3, 1.4, 1.5 and 1.5 to find the second focal length for a double-convex lens with the following specifications and media. $|R_1| = 6.58cm$, $|R_2| = -70.3cm$. Use the "Gaussian" sign convention. Assume the index of refraction for the lens is $n = 1.5$.

   a) Air on both sides of the lens.

   b) Water on both sides of the lens

[3] p75, Jenkins and White, Fundamentals of Optics, 4th Edition
[4] p81 of Jenkins & White, Fundamentals of Optics, 4th Ed.
[5] Wes Wong Optics Lab manual, Experiment 2, p10

c) air on the first (left) side and water on the second (right) side.

d) Water on the left side and air on the right side of the lens.

1.5 Procedure

1.5.1 Hollow Lens Behavior

A hollow lens has three sections as shown in figure 1.2:

1. a plano-concave section
2. a plano-convex section with a small R value near $|R| \approx 30mm$
3. a plano-convex section of larger R value near $|R| \approx 48mm$

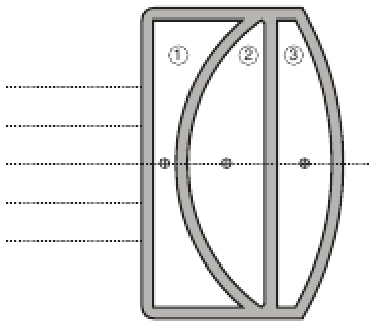

Figure 1.2: The hollow lens apparatus

1. Place the hollow lens apparatus in front of the light ray source with the flat side closest to the source. Select five or three rays, whichever performs best. The sections will be alternately contain air or water according to a table shown later.
2. For testing with water surrounding the lens, place the lens inside the plastic rectangular box with the white plastic sheet on the bottom as shown in Figure 1.3. The apparatus should nearly be submerged so the rays from the source are transversing the same situations in terms of index of refraction. Place coins on top of the lens to keep it from floating as necessary.
3. Create the required combinations for what each section has according to Table 1.1 Fill in the table with your observations as to whether a diverging or converging lens is created.

1.5.2 Thick Lens

1. Remove the lens from the plastic box and rotate it so that section 1 faces the light[6] and place the ray source and lens apparatus on a large sheet of paper.
2. Trace around the lens footprint so that its location is shown on the paper.
3. Fill section 2 and 3 with water to make a double-convex lens. Section 1 should have air in it. Trace three rays to their focal point.
4. Measure the focal length of this water dbl-cvx lens in mm using your best guess as to the center of the lens:

$$f'_{dbl-cx} = \qquad\qquad mm$$

[6] We want a flat surface facing the light source.

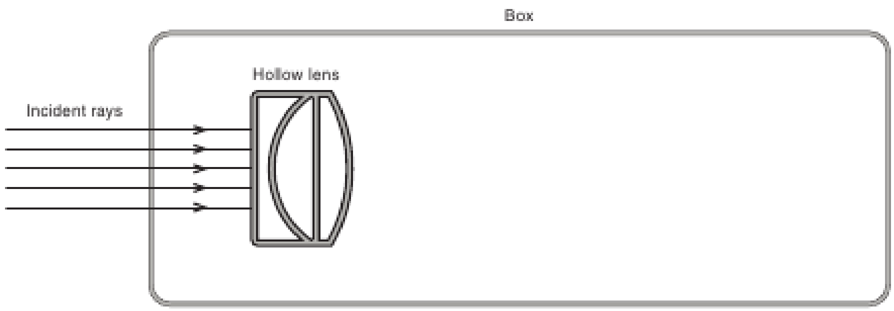

Figure 1.3: Hollow lens in box for surrounding it with water

1.6 Results

Table 1.1: Observations

Lens surrounded by	fill section 1 with	fill section 2 with	fill section 3 with	Lens type C = converging or D = diverging
air	water	air	air	
	air	water	air	
	air	air	water	
	water	air	water	
water	air	water	water	
	water	air	water	
	water	water	air	

1.7 Questions and Calculations

1. Under what conditions is the is the plano-convex lens converging?

2. Under what conditions is the plano-convex lens diverging?

3. If a plano-concave lens of unknown material is a diverging lens when surrounded by air, is it possible to know whether the lens will be converging or diverging when placed in water? Explain why or why not.

4. The double-convex lens in this experiment has $R_1 = 30mm \pm 3mm$ and $R_2 = 48mm \pm 3mm$.

   a) Use Equation 1.6 to calculate the focal length for the double-convex lens measured in subsection 1.5.2.

   b) Is it appropriate to use 1.6 this formula for this situation? Explain why or why not.

   c) Use equation 1.7 to calculate the focal length of the thick lens. The thickness for the double convex lens in this case is $d = 25mm$. Was there better agreement with the measured focal length?