

11. Single and Double Slit Diffraction

11.1. Purpose

To compare the predicted single + double slit interference patterns (intensity as a function of linear position) to data collected for the same.

11.2. Theory

11.2.1. Single Slit Equation

The relevant theory is developed in numerous texts. Only the final results are quoted here with a description of each parameter. The variables are:

- θ , the angle of deviated the rays which have been diffracted by the slit
- b : the width of a single slit
- d : the separation between adjacent slits
- λ : the wavelength of the light source (laser)
- b : the width of each slit for a double slit setup
- x : linear distance on a screen
- s : distance from slit(s) to screen

For the single slit, the intensity $I(\theta)$ of light on the screen will follow equation 11.1.

$$I(\theta) = I_0 \frac{\sin^2 \beta}{\beta^2} \quad (11.1)$$

where $\beta = \frac{\pi b \sin \theta}{\lambda}$

For small angles of θ , the linear position on the screen will scale in the same way (doubling θ will double x), so to a good approximation:

$$I(x) = I_0 \frac{\sin^2 \beta}{\beta^2} \quad (11.2)$$

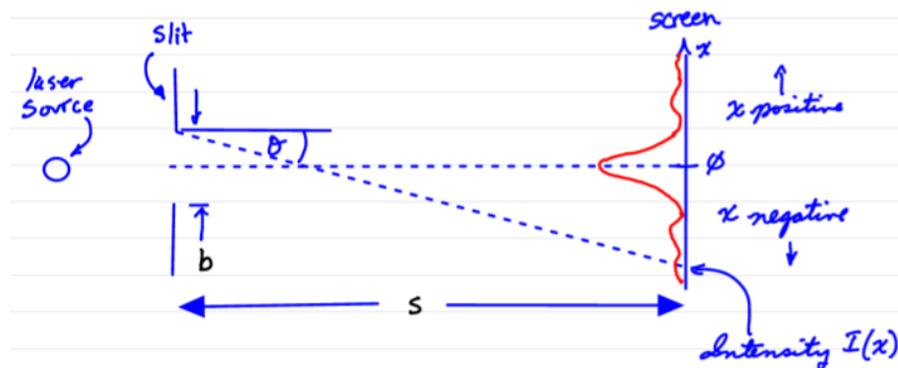
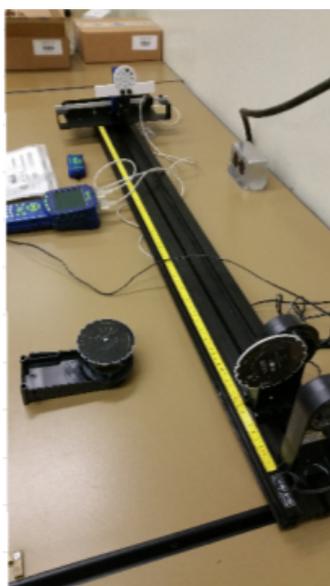
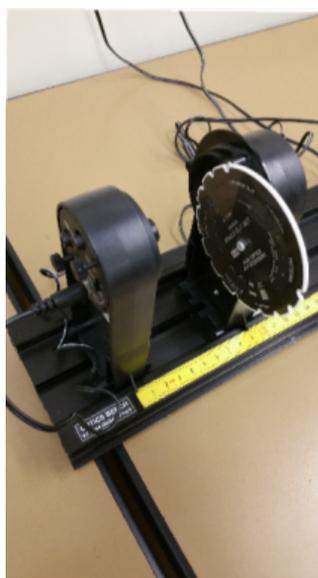


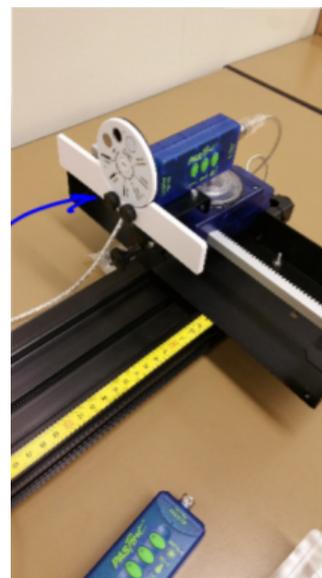
Figure 11.1.: Single slit variables



(a) Complete Apparatus



(b) Laser Source & Slit Accessory



(c) Aperture Bracket, Light Detector and Linear Translator

Figure 11.2.: Setup of apparatus

which is what we will measure. Please see Figure 11.1 which shows the intensity as a function of position as a red line.

11.2.2. Double Slit Equation

If a double slit is used, the relevant intensity function is

$$I(x) = 4I_0 \frac{\sin^2 \beta}{\beta^2} \cdot \cos^2 \alpha \quad (11.3)$$

where $\alpha = \frac{\pi d \sin \theta}{\lambda}$ and β remains the same as in the single slit equation.

Note that this formula merely imposes the multipliers 4 and $\cos^2 \alpha$ onto the single slit formula and that $d > b$ or the two slits become one slit.

11.3. Apparatus

Pasco laser diode, 1.2m optics bench, single slit accessory, double slit accessory, regular or high sensitivity light sensor, rotary motion sensor, linear translator, aperture bracket.

11.4. Procedure

11.4.1. Mounting Components on the Bench.

Set the equipment up as shown in Figure 11.2. Table 11.1 has positions which yield decent results. See the Figure 11.2 for more detail.

The slit accessory uses a mount which allows rotation and translation due to two offset axes of rotation. This behavior is both useful and sometimes detrimental in that the axis you don't want to move does. Examine the slit accessory device to understand its construction.

laser source	0cm
slit accessory	7cm
aperture bracket	100cm

Table 11.1.: Suggested equipment positions

Figure 11.3.: Single slit pattern projected horizontally onto aperture bracket



The orientation of the linear translator requires a diffraction pattern which spreads out *horizontally* as shown in Figure 11.3. Rotate the slit accessory for height and alignment to achieve a pattern that is horizontal and centered on the slit in front of the light sensor.

11.4.2. Choice of Slits

The choice of slit on the slit accessory wheel sets the slit width a . Determine what slit width a gives a bright and easily seen pattern.

The aperture wheel (on its “bracket”) in front of the light sensor helps “choose” the section of the pattern detected. This relates to resolution of the peaks and valleys. This second slit is chosen to give decent sensitivity to changes in detected light intensity without saturating the detector readings (flat top).

The distance s is measure as the distance in between the slit wheels.

Table 11.2.: Distances and Settings

Pattern	slit width b	slit separation d	aperture slit	s	Light sensor range ¹
single slit		na			
double slit					

Record the required values in Table 11.2.

11.4.3. Procedure: Taking Data

After getting a reasonably bright and level pattern projected onto the slit aperture, record a pattern.

1. Data collection must start *slightly* to the left or right of the pattern, not somewhere in the pattern, that is, start just to the outside a faint maxima.
2. Press record on the GLX and slowly turn the pulley wheel to make the light sensor translate across the pattern.
3. Steadily move the sensor. Practice this until you record a reasonably good pattern.

11.4.4. Double Slit Pattern

Switch to the double slit accessory wheel. Record b , d , s . Laser $\lambda = 650nm$

Suggested double slit: slit width $b = 0.08mm$ slit separation $d = 0.25mm$, but more interesting results can be achieved.

¹button on light sensor

11.5. Analysis

The analysis consists of two main steps:

1. Plot the sensor data.
2. Plot the theoretical equation over the data and compare.

11.5.1. Plotting Sensor Data

11.5.1.1. Exporting Data from GLX to USB memory

1. Export the best single slit run and the best double slit run from the GLX to a USB stick: Home, Tables, check-mark, Arrow to Run#, check-mark, choose run to export. Populate two columns with linear position and light intensity.
2. This will create a text file with three columns; the row index number will be in the left-most column. Importing into xmgrace will involve choosing only second and third columns.
3. Take the USB stick to a lab machine and copy the data to a folder in your home directory.
4. View the data with “pluma” or other text editor so that you understand where the columns of data are in the file.

11.5.1.2. Single Slit: Import data to xmgrace

Import the data into xmgrace using the following instructions concerning menu entries,etc:

- Data, Import, ASCII, Load as: Block Data, OK. Choose the columns which have:
 1. linear position² for X
 2. and light intensity for Y.

Now that the data is available as a set in xmgrace, eg g0.s0 with x as linear position and y as the light intensity, a comparison to the theory can be made by plotting the theoretical equation on top of the data. First the following corrections must be applied.

11.5.1.3. Corrections to Linear Position

Corrections to the data must be applied:

- offset removed,
- correcting linear scale (distance).

11.5.1.4. Offset Correction

Your data will not centered about $x = 0$, but the theoretical equation assumes zero is at the center and at the maximum intensity is located there. A correction to the observed data is one strategy. This means shifting the observed data so that the central maximum is at $x = 0m$ as shown roughly in Figure 11.4



Figure 11.4.: center data about zero on x-axis

²Not the radian position of the RMS sensor’s wheel/gear

NB: As you add and hide sets, make sure you label them using the comment field and turn on the comments. Also use the “Pack all sets” feature which will relabel the sets to be sequential if you delete a previous set. That will sync up your sets with the description/labels shown in these instructions and reduce confusion.

1. Center the observed peak to zero linear displacement by creating a new set in xmgrace which is centered on zero: Scan down the data in the xmgrace’s table g0.s0 and locate the maximum light intensity value.
2. Add or subtract this offset to the linear position of a new set eg. g0.sl.x generated with Create new, By formula. Y will be the same values as before: $Y=g0.s0.y$
3. Hide the original set, scale and display this new set.
4. Label the new set: “centered peak” in the set comment field and display the set comments.

11.5.1.5. Horizontal Position Scale Correction³

This step is not necessary if “rack” is available in the options for linear position of the RMS sensor’s output.

The output of the RMS sensor can set to linear position, which does an internal conversion from angular displacement to linear displacement using an assumed radius or circumference. If the GLX does not offer the selection of the internal gear of the RMS sensor, a scale correction is necessary. Thus the RMS data is proportional to linear displacement, not equal to it. *The following is for the default linear setting of the GLX/RMS combination.* This is quickly drawn in Figure 11.4.



Figure 11.5.: Scale correction for horizontal position

1. For a true distance of 0.182 m, the RMS linear distance will be 0.341 m, a value that is too large, thus the x-axis must be scaled down by $0.182\text{m}/0.341\text{m}$. Create a new set (g0.s2) where $X = g0.sl.x * 0.182/0.341$.
2. Use $Y = g0.sl.y$ to copy over the Intensity values without change.
3. Hide the old .sl set, auto scale and display the new set.
4. Label the new set: scaled and centered X

11.5.2. Plotting the Theoretical Equation

Plotting the theoretical equation $I(x) = I_0 \frac{\sin^2 \beta}{\beta^2}$ over the data involves the following steps:

- Determine what θ is equal to in terms of x and s .
- Formulate β in terms of x and s .
- calculate β
- calculate $I(x) = I_0 \frac{\sin^2 \beta}{\beta^2}$

³If linear position of the internal gear is not available in the RMS sensor’s setup

11.5.2.1. Remove zeros, calculate β

The calculation of the intensity pattern requires dividing by $\beta = \frac{\pi b \sin \theta}{\lambda}$. If x goes to zero then β does too and the software can't plot $I(x) = I_0 \frac{\sin^2 \beta}{\beta^2}$ due to a divide by zero condition. Use the following steps to produce the theoretical plot:

1. Duplicate the “scaled and centered” set. This is done by selecting the set, right clicking on it, Duplicate.
2. Display the newly created set and delete rows near the middle of the set where $X = 0$. Label this set “no zeros”.
3. Calculate a set for $\beta = \frac{\pi b \sin \theta}{\lambda}$:
 - a) Determine what θ is in terms of x and s . Refer to this as $f(x, s)$.
 - b) Calculate the value of $\frac{\pi b}{\lambda}$ by calculator and insure that units cancel.
 - c) Create a new set by calculation for $\beta = \frac{\pi b}{\lambda} \cdot f(x, s)$ where the function is your equivalent to $\sin \theta$. Label this set “beta”.

11.5.2.2. Calculate $I(x) = I_0 \frac{\sin^2 \beta}{\beta^2}$

1. Open the old set that still has the $X=0$ data in it and obtain your value for I_0 which is the maximum intensity value observed. It will be near $X=0$.
2. Calculate a new set for $I(x)$. For example if I_0 is 3.48 lux and g0.s5.x has the β values in its Y column, the new set would have: $X = g0.s5.x$ and $Y = 3.48 * (\sin(g0.s5.y))^2 / g0.s5.y^2$

At this point you hopefully have a predicted single slit pattern line plotted which agrees very well with the data collected.

Present the data as circles and the theoretical line as a line or dashed line in xmgrace. For the report just the corrected data and the prediction line need to appear. See Figure 11.6 for an example where extra information is shown. It was collected to insure the slit width and Intensity values, both observed and predicted, matched well.

11.5.3. Import Double Slit Data

Use the Block data import feature of xmgrace to import the double slit data. An example is shown in Figure 11.7. The dashed line is not a theoretical fit but merely a dashed line joining the data points.

11.5.4. Question:

What further step would be required to predict the double slit pattern with reference to the steps taken to plot the single slit pattern?

Single Slit Diffraction

0.08mm slit, 93cm slit to screen, 650nm laser

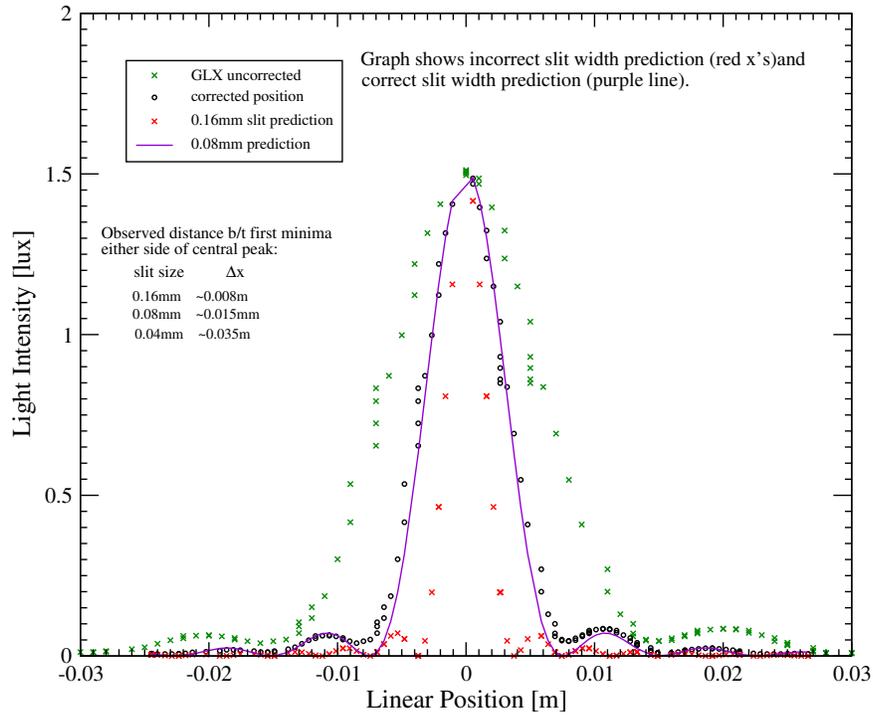


Figure 11.6.: Single Slit graph: data and predicted curve.

Example Data: Double Slit Diffraction

$a = 0.08\text{mm}$, $d = 0.25\text{mm}$, $\lambda = 650\text{nm}$

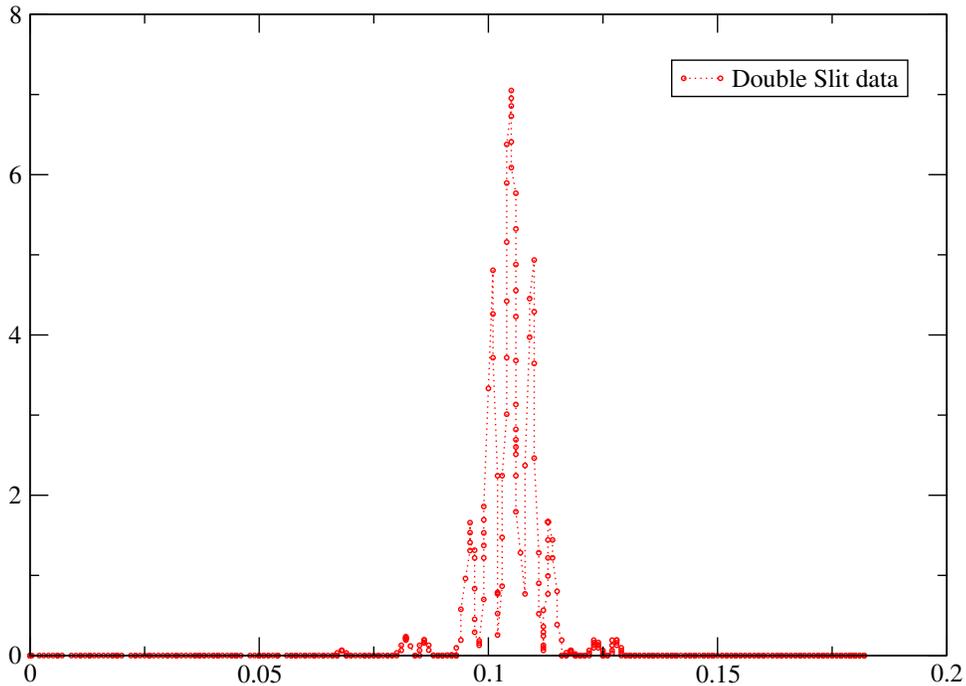


Figure 11.7.: Double Slit data

11.6. Hints and Help

- Turn back-light on: Home, Settings, arrow down to back-light, check-mark, choose ON.
- It is very useful to label your sets in xmgrace. For the top figure on page 9, the following sets were present in xmgrace, just as an example:
 - G0.S0 [2][149] "original data"
 - G0.S1 [2][149] "data centered at zero" (moved data; peak I@0m)
 - G0.S2 [2][149] "scaled and centered X" (corrected for scale 0.182/0.341)
 - G0.S3 [2][144] "no zeros, copy of G ".S2" (removed x=0 data points)
 - G0.S4 [2][144] "0.16mm Beta calculation" ($\beta = \frac{\pi b}{\lambda} \sin\theta$ calc)
 - G0.S5 [2][144] "0.16mm prediction" (predicted curve b = 0.16mm)
 - G0.S6 [2][144] "0.08mm Beta calculation" ($\beta = \frac{\pi b}{\lambda} \sin\theta$ calc)
 - G0.S7 [2][144] "0.08mm prediction" (predicted curve b=0.08mm)
- Radians: If the GLX data was not exported in meters of linear position, it is probably in radians. The metric is actually the RMS pulley's position in radians, not the angle of interest θ .