Blackbody Radiation

Introduction:

file: Blackbody Radiation Instructions.pdf on lake_mountain 2023 copied from 2019

This is one of the key experiments that led to the development of Quantum Mechanics.

The spectrum of an incandescent light bulb is scanned by hand using a prism spectrophotometer that measures relative light intensity as a function of angle. A Broad Spectrum Light Sensor is used with a prism so the entire spectrum from approximately 400 nm to 2500 nm can be scanned without th overlapping orders caused by a grating. The wavelengths corresponding to the angles are calculated using the equations for a prism spectrophotometer. The relative light intensity can then be plotted as a function of wavelength as the spectrum is scanned, resulting in the characteristic blackbody curve. The intensity of the light bulb is increased, increasing the temperature, and the scan is repeated to show how the curves nest with a shift in the peak wavelength.

The temperature of the filament of the bulb can be estimated indirectly by determining the resistance of the bulb from the measured voltage and current. From the temperature, the theoretical peak wavelength can be calculated and compared to the measured peak wavelength.

This experiment should be performed in a room with reduced light levels although complete darkness is not required.

Written by Chuck Hunt

Equipment:

	INCLUDED:	
1	Prism Spectrophotometer Kit	OS-8544
1	Optics Bench (60 cm)	OS-8541
1	Spectrophotometer Accessory Kit	OS-8537
1	Aperture Bracket	OS-8534B
1	Broad Spectrum Light Sensor	PS-2150
1	Rotary Motion Sensor	PS-2120A
1	Voltage Sensor	UI-5100
1	Replacement Bulb (10 pk)	SE-8509
1	Banana Plug Cord-Black (5 pack)	SE-9751
NOT INCLUDED, BUT REQUIRED:		
~~1~~	~~850 Universal Interface~~	~~UI-5000~~
~~1~~	~~PASCO Capstone~~	

1 Pasco GLX data logger

This document is the result of adapting a Pasco lab from a Capstone lab to the use of a Pasco GLX, xmgrace for nonlinear curve fitting and a brighter light source. Information regarding the deviation from ideal BB data is revealed and the tungsten filament temperature is found more than one way using the data.

Purpose: 1) To capture the blackbody curve of a tungsten filament at various temperatures.

2) To fit the blackbody equation to the data and verify features of the black body as temperature changes.

3) To determine one or more temperatures of the tungsten filament.

Theory:

The intensity per wavelength, $I_\lambda(\lambda,T)$, as a function of wavelength of radiation emitted by an ideal body (a blackbody since a ideal emitter must also be an ideal absorber) is given by Planck's Radiation Law:

$$I_\lambda(\lambda,T)=\frac{2\pi c^2 h}{\lambda^5}\left(\frac{1}{e^{(hc/\lambda kT)}-1}\right) \qquad (1)$$

where c is the speed of light in a vacuum, h is Planck's constant, k is Boltzmann's constant, T is the absolute temperature of the body, and λ is the wavelength of the radiation. Any real object must emit less at all wavelengths.

The wavelength with the greatest intensity is given by

$$\lambda_{max} = (\text{constant})/T = (0.002898\ \text{m*K})/T \qquad (2)$$

The temperature of the blackbody light filament can be calculated using the resistance of the filament while it is lit. We find the resistance (R) by measuring the voltage (V) and the current (I) and using R = V/I. The resistance of the tungsten filamei is a nonlinear function of the temperature. Using the measured resistance to calculate the temperature is discussed in Appendix ~~1~~. 2

The wavelength is determined by measuring the angle at which the light is dispersed by a prism. The relationship between the angle and the wavelength is discussed in Appendix ~~2~~. 1

Appendix #'s are corrected

THEORY

The following equations pertain to BB radiation curves, etc.

$\lambda_{max}\cdot T = 2.898e\text{-}3\ m\cdot K$ (peak wavelength)

Energy per unit volume per wavelength $S_\lambda \left[\frac{J}{m^3\cdot m}\right]$

$$S_\lambda = \frac{8\pi h c}{\lambda^5}\cdot\frac{1}{e^{hc/\lambda kT}-1}$$

T in kelvin [K]

"red hot" → 900 - 1000 K
tungsten filament ↙ 2k - 3k K
or 2550 °C ↙
75 W to 100 W

1 W = 1 J/s

Radiation Power Density $\left[W/m^3\right]$

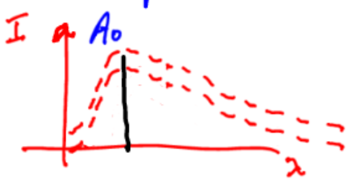
$$S(\lambda) = \frac{2\pi c^2 h}{\lambda^5}\cdot\frac{1}{e^{\frac{hc}{\lambda kT}}-1}$$

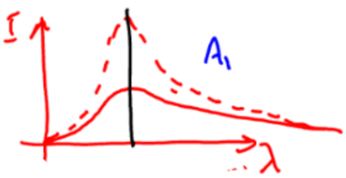
The equation for fitting in xmgrace: Three parameter fit:

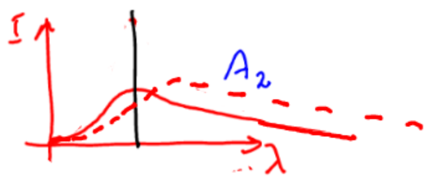
① $A_0 + (\quad)$ ② $A_0 + A_1/\lambda^5\cdot(\quad)$ ③ $A_0 + \frac{A_1}{\lambda^5}\cdot\frac{1}{e^{A_2/\lambda}-1}$

use 3 parameter fit

A_0 Offset "term" A_1 "Amplitude" Term A_2 "Shape" term has T dependence







Light Source: replace with desk lamp due to these problems:

1) BB light source bulbs easily burned out
2) " " " is dim.

Task: Please calculate the constants below (MKS)

Energy per unit volume per wavelength $S_\lambda \left[\frac{J}{m^3 \cdot m}\right]$

① $S_\lambda = \frac{8\pi hc}{\lambda^5} \cdot \frac{1}{e^{hc/\lambda kT} - 1}$ } energy density per λ

Radiation Power Density $\left[W/m^3\right]$ and $1\,W = 1\,\frac{\text{Joule}}{\text{second}}$

② $S(\lambda) = \frac{2\pi c^2 h}{\lambda^5} \cdot \frac{1}{e^{\frac{hc}{\lambda kT}} - 1}$ } power density per λ

From hyperphysics online: To find the radiated power per unit area from a surface at this temperature, multiply the energy density by c/4. There are links explaining the c/4 factor extensively.

We will fit to (②)

S is used for both formulas above → caution needed ¡

① $2\pi c^2 h =$

② $hc =$

③ $hc/k =$

① relates to A_1 in fit   ③ relates to A_2 in fit

The following two pages develop the equations required to convert Table angle to wavelength λ. Our data is aquired with the angle the radiation leaves the prism at encoded by a rotary motion sensor (RMS). This is directly related to actual Table angle linearly. Since the index of refraction has a λ dependence, a "prism" equation can be developed that relates refracted angle to wavelength λ.

Appendix 1: Finding the wavelength as a function of angle.

The index of refraction of the prism glass varies with the wavelength of the light. To determine the wavelength as a function of the angle, the relationship between the index of refraction and the angle is determined using Snell's Law at each face of the prism and some geometry and basic trigonometry.

$\sin 60^0 = n \sin \theta_2$ (A1) and $\sin \theta = n \sin \theta_3$ (A2) ← Snell's law A_1 + A_2 equations

where n is the index of refraction of the prism.

$n \sin \theta_3 = n \sin (60^0 - \theta_2) = n(\sin 60^0 \cos \theta_2 - \cos 60^0 \sin \theta_2)$

$= n \sin 60^0 \cos \theta_2 - \cos 60^0 \sin 60^0$ (using Equation A1)

Rearranging this and using Equation A2 yields

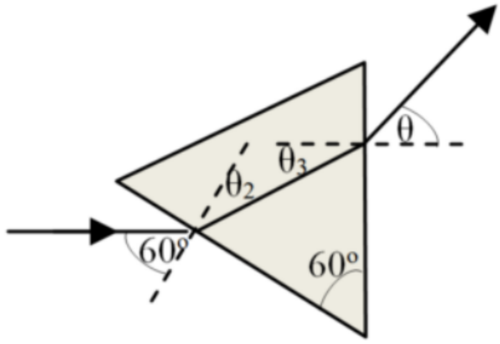
$n \cos \theta_2 = (\sin \theta / \sin 60^0) + \sin 60^0$ (A3)

Squaring Equations A1 and A3 and adding them to gives

$n^2 (\sin^2 \theta_2 + \cos^2 \theta_2) = n^2 = [(\sin \theta / \sin 60^0) + \sin 60^0]^2 + \sin^2 60^0$

Putting in values for sin 60⁰ and cos 60⁰ yields

$$n = \sqrt{\left(\frac{2}{\sqrt{3}} \sin\theta + \frac{1}{2}\right)^2 + \frac{3}{4}} \qquad \text{(A4)}$$

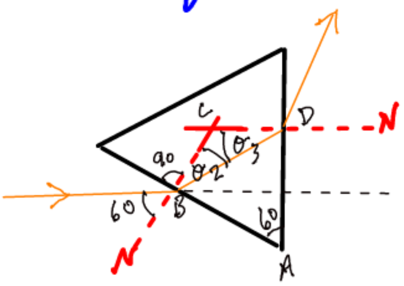


We use this equation to calculate index of refraction (n) values for our measured angles. We then use the n values to calculate the wavelength using values relating the index of refraction to wavelength for the prism (provided by the supplier of the prism) (see Table III under tab Append 1B).

A4 is n as a function of sinθ : n(sinθ) (not n × sinθ)

n(sin θ) goes into eq'n on next page to get Intensity % as a function of λ

Checking derivation



$$n = \sqrt{\left(\frac{2}{\sqrt{3}} \sin\theta + \frac{1}{2}\right)^2 + \frac{3}{4}} \qquad \text{(A4)}$$

Despite error in derivation, the correct #, the ½, was put into formula under the √.

On the next page, we have eq'n A5: λ as a function of n

Note the graph below has a range of λ from 400nm to ~2500 nm. The equation developed (A5) applies to this range of λ and not much outside it. This also means the range of Table angle is similarly limited in range ⇒ the data acquired will need truncation. (more on this on next page)

Table III: Prism Data

	Index of Refraction	Lambda (nm)
1	1.68	2325.4
2	1.69	1970.1
3	1.69	1529.6
4	1.70	1060.0
5	1.70	1014.0
6	1.71	852.1
7	1.72	706.5
8	1.72	656.3

We need an equation based on the data in the table to use i our calculations. We use a polynomial and choose the value of the constants to fit the prism data. The results are not unique but fit the data within the uncertainty in the index implicit in the data table of at least 0.005. The equation is

λ =

$$l = A + B(n-E)^{1} + C(n-E)^{-2} + D(n-E)^{-3} \qquad (A5)$$

where l is the wavelength in nm, and the constants have values: A = 320 nm, B = 1 nm, C = 0.2 nm, D = 0.19 nm, and E = 1.635. On the graph, the circles represent the prism supplier's data with the size of the circles showing the uncertainty, and the curved line is from the above equation.

now put
$$n = \sqrt{\left(\frac{2}{\sqrt{3}}\sin\theta + \frac{1}{2}\right)^2 + \frac{3}{4}} \qquad (A4)$$
into equation A5

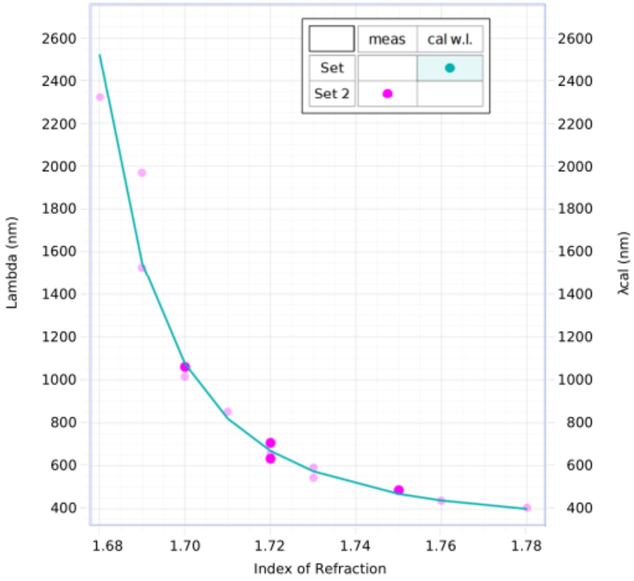


Figure 11: Prism Wavelength vs. Index of Refraction

So we have finally λ as a function of angle (or sinθ):

λ = 320 nm +

tells xmgrace angle is in degrees not radians

1nm*(sqrt((1.1547*sin(90.5#.x deg)+0.5)^2+0.75) − 1.635)^-1

+ 0.2nm*(sqrt((1.1547*sin(90.5#.x deg)+0.5)^2+0.75) − 1.635)^-2

+ 0.19nm*(sqrt((1.1547*sin(90.5#.x deg)+0.5)^2+0.75) − 1.635)^-3

This is entered into xmgrace to convert the x-axis to λ [nm] from Table Angle in degrees. So we move from ② to ③

METHOD

If the apparatus is not set up, use the following instructions.

RMS = Rotary Motion Sensor:

1. Remove black mounting h/w from RMS sensor using provided hex-key. Store hex key in RMS for later use.
2. Remove 3-step pulley + replace knurled retaining screw onto RMS shaft.
3. RMS to spring-loaded plate:
   a) RMS orientation: curved arrow showing + ↗ is upward.
   a) Use lower pair of holes: gives proper distance and alignment for pinion: small radius portion engages table.
   b) May require 2 small washers as mounting screws could bottom-out in RMS.
4. See further instructions and diagram below.

Spectrophotometer Set Up

This part of the manual describes how to set up the Spectrophotometer System (see Fig. 3).

Mounting the Rotary Motion Sensor

This describes how to mount the Rotary Motion Sensor to the hinge on the Spectrophotometer Base. The top of the Spectrophotometer Base has a short threaded post for centering the circular Degree Plate and for holding the Grating Mount. It also has a magnetic pad for holding the Degree Plate, and a triangular shaped index marker. One side of the base has a post upon which the Pinion can be stored when it is not in use. The other side has a spring-loaded hinge and two small thumbscrews for mounting the Rotary Motion Sensor (included in the Spectrophotometer System). On both sides of the base are large thumbscrews and square nuts used for mounting the Spectrophotometer Base on the Optics Bench (see Fig. 4). The Rotary Motion Sensor has a three step pulley attached to its shaft with a small thumbscrew. The sensor also has a rod clamp attached a one end. First, remove the small thumbscrew and three step pulley from the Rotary Motion Sensor shaft. Then, remove the rod clamp from the Rotary Motion Sensor (see Fig. 5).

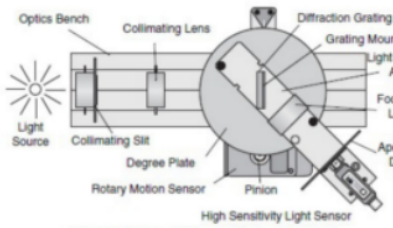


Figure 3: Spectrophotometer System (top view)

X prism arrow shown on wrong side

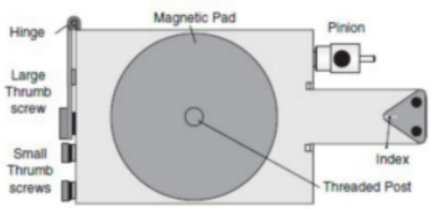


Figure 4: Spectrophotometer Base (top view)

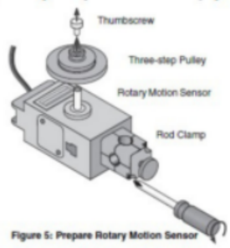
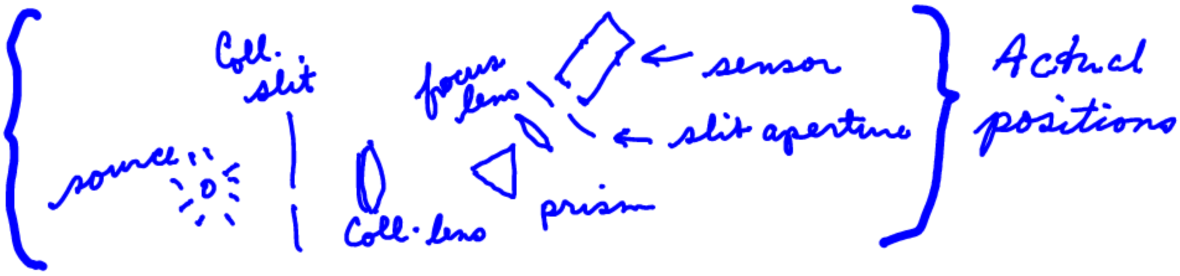


Figure 5: Prepare Rotary Motion Sensor

look for curved arrow indicating direction of increasing angle and put this side of the RMS upward. →



Setup A

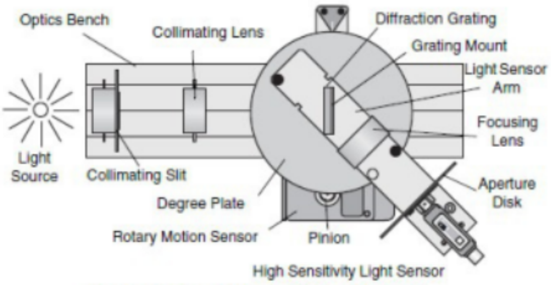


Figure 3: Spectrophotometer System (top view)

Figure 1: Complete Setup

1. Set up the Prism Spectrophotometer as shown in Figure 1 except place the Blackbody Light Source close to the left end of the track and the Collimating Slit closer to it than is shown in the picture to maximize the intensity. Detailed instructions for mounting the Rotary Motion Sensor and the Degree Plate and Light Sensor Arm to the spectroscopy table may be found in Appendix 3.

2. Attach the Broad Spectrum Light Sensor to the Light Sensor Arm using the ½ inch bolt ($^1/_4$ x 20) with a large black plastic head. Attach the 2 inch black rod to the bottom of the Light Sensor Arm using one of the vacant holes. This makes a convenient handle for sweeping through the spectrum (see fig. 1 above).

3. Mount the Beveled Stop Piece on the bottom of the Light Sensor Arm with the two supplied bolts (see Figure 2). Position the beveled edge so it will hit against the angle indicator on the spectroscopy table.

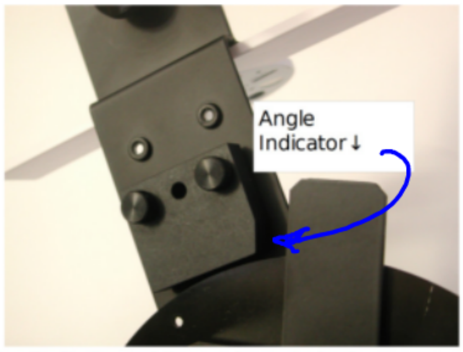


Fig. 2: Bottom View - Beveled Stop

1. Attach the Mounted Prism to the spectroscopy table by screwing it into the hole in the center of the table. Screw it down until it almost touches the table. is critical that the Mounted Prism does not touch the table so the table is free t move without moving the prism. Orient the prism with its apex toward the ligh source as shown in Figure 4. The prism base must be perpendicular to the incoming light beam. To do this set, turn the table until the index mark is on 0' and then set the base of the Mounted Prism so it lies along the 0°-180° line on the table. Secure the prism in place using the wing nut and lock washer on the bottom of the bolt sticking through the spectroscopy table.

2. Ground the Spectrometer by attaching an alligator jumper cable from the ground post on the bottom of the spectroscopy table (on the side opposite the Rotary Motion Sensor), and attaching the other end to a ground. A convenient ground is the silver outside connector for the #2 or #3 Outputs at the lower right on the 850 Universal Interface.

3. Plug the Blackbody Light Source into the #1 Output on the top right of the 850 Universal Interface. Polarity does not matter.

4. Plug the Broad Spectrum Light Sensor and the Rotary Motion Sensor into PasPort inputs on the 850 Universal Interface.

5. Plug the Voltage Sensor into the Analog A input (see Figure 1). Attach the red lead to the red banana lead on the Blackbody Light Source. Attach the blac lead to the black banana on the Blackbody Light Source. Do Not attach the lea to the output jacks on the 850. The current is large enough that there is a voltage drop along the wires connecting the 850 output to the Blackbody Light Source and we want to measure the voltage at the light source.

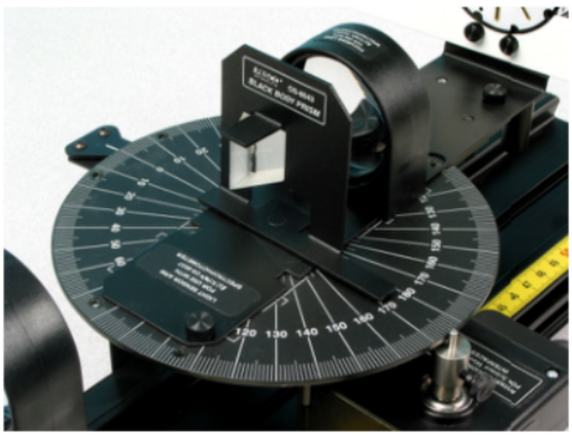


Figure 4: Prism Orientation

Figure 1: Complete Setup

(5) Alignment: NB

* Once constructed one must know that 0° is really 0°. This means location of:
  - light source
  - collimating slits
  - collimating lens
  - prism
  - focusing lens
  - slit aperture in front of broad spectrum sensor
  - broad spectrum sensor.

must all be positioned so that the "white" light from the source hits the slit aperture when the table is set to zero degrees.

* An effective way to move the beam of light for this step is to slightly loosen + the collimating slits and watch where the light lands on the slit aperture

Procedure

1. Set the collimating slits on Slit #4. Set the Light Sensor mask on Slit #4. See Figure 3.

2. Collimating the system: the Collimating Slit must be at the focal point of the first lens and the Senso Mask and Aperture Disk must be at the focal point of the second lens. Move the spectroscopy table bac to the end of the track it is out of the way. Place the Blackbody Light Source near the end of the track and the Collimating Slit near the blackbody light source. Move the Collimating Lens (see figure 3 above at least 12 cm from the slit. Have someone with 20/20 vision (corrected by glasses is fine) look througł the lens at the slit. Move the lens toward the slit until it first comes into sharp focus. The slit should be about 10 cm from the lens. Now move the spectroscopy table as close to the Collimating Lens as possible. Set the Focusing lens 10 cm from the Sensor Mask. We will adjust this more exactly later.

3. Click the Signal Generator at the left of the screen. Set the waveform for DC and the voltage for 7.0 either by typing it in or by using the up/down keys to the right of the DC Voltage bar. Turn on the Signa Generator by clicking ON.

4. Set the moveable arm at the center of the track so that the un-deviated light that passes above the prism from the slit strikes the Sensor Mask. Adjust the Focusing Lens so the image on the Sensor Mask is as sharp as possible. The system is now well collimated. Look at the light coming from the Blackbody Light Source. Observe the color. (Yes, white is a color.)

5. Rotate the movable arm until you see the spectrum. Look at the spectrum on the Light Sensor screen. Are all the colors (from red to violet) present? What does this show about white light?

6. Rotate the scanning arm until it touches the stop. This will be the starting position for all the scans.

7. Read step 8 before you do this step! Click RECORD (at bottom of the page). You must be holding th scanning arm against the stop when you press RECORD!!! If it is not against the stop, each run will hav a different zero position and you will not see the position of the peak correctly.

8. The Broad Spectrum Light Sensor tends to drift so the following steps need to be performed as written. With the sensor are pressed against the stop, press the Tare button on the Broad Spectrum Light Sensor (the button is illuminated) to zero it. Observe the width of the visible pattern. Using the handle below the light sensor, sweep rapidly from the stop to a position about one visible spectrum wid to the left (ultraviolet side) of the pattern. Slowly rotate the scanning arm through the spectrum to poi about two visible spectrum widths to the right (infrared side) of the visible pattern. You should try to complete this operation in less than 30 seconds from when you press the Tare button, but try to sweep at a uniform rate. Now continue rapidly all the way past zero degrees (the position where the light sens is directly opposite the light source), slowing as you sweep across the white light peak at zero degrees. is important that you only sweep in one direction! If you attempt to go back, the Rotary Motion sensor will lose track of where you are!

9. Click Stop. On the Signal Generator, click Off. Click on the Signal Generator Button to close the

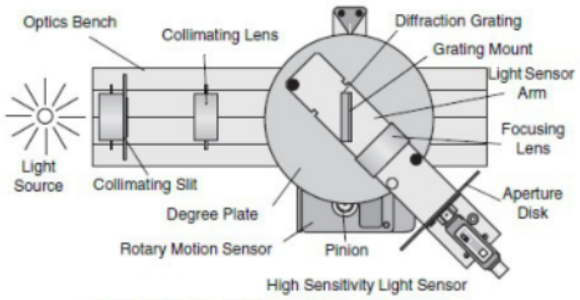


Figure 3: Spectrophotometer System (top view)

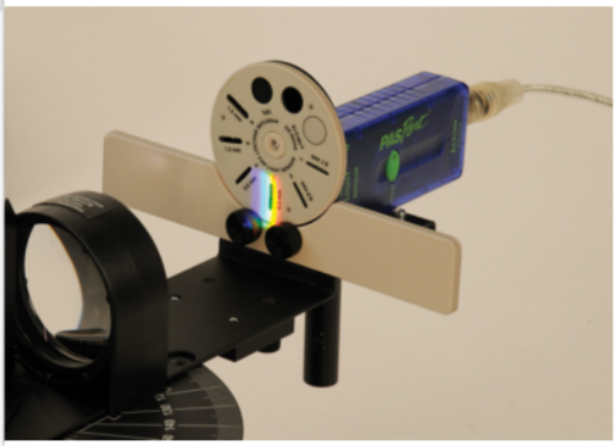


Fig. 5: Spectrum on Light Sensor Mask

* 30 second time suggested above : avoid sensor drift issue. This is easily accomplished.

METHOD

① NB: Sample rate for both sensors (RMS + broad spectrum light sensor) must be set to:

40 samples/sec

Use HOME, Sensors, ✓ + change sample rate to the above value.

② A demonstration of the data collection will be given.

③ Please use the File, Save feature on the GLX before exporting your data to USB memory.

METHOD

Angle correction:

We need the ability to convert RMS angle to table angle since table angle is the θ used in n(sinθ) formula. To accomplish this.

① Put GLX in digits display mode (available on HOME screen).

② Display RMS angle to 3 or 4 digits.

③ Start at table angle zero + press record/run

④ Record RMS angle. Increment table angle by 10° + record RMS angle. Keep going until stop is reached
(↑ physical).

RMS angle = X   Table angle = Y

⑤ Plot Table Angle as a function of RMS angle + do a linear fit in xmgrace. The equation of fit can then be used to create a new set of intensity as a function of table angle.

METHOD Lamp Voltages + Runs

* Use the variac + start lamp @ max brightness ~130V_{rms}

* Do four voltages:

	130V	Perform angle sweeps in the same direction as your table + RMS data was collected.
	110V	
	85V	
	60V	

NB: keep track of Run# and voltage

† { * Use the GLX file manage to copy the file to a USB stick.

* Then export each run to USB stick. Use a nearly empty stick. }
  - This is done in the Table display on the GLX.
  - Select the Run to be displayed in the table, press F4, 8, ↓ down arrow, ✓ check to change UniCode to ASCII, F1 = ok.

† these are not the same thing or process.

Truncating data will likely need to be done to get sensible graphs for %I as a function of Table Angle [degrees]. This happens because we can swing the table to angles for which the prism + prism formula don't refract or function. Thus the apparatus can be set to angles past the infrared; the prism will not function at those supposed λ values.

ANALYSIS

① Import data in xmgrace as block data with X from Column 2 and Y from Column 3. Column 1 in the txt file will be the sample # (integer) and is not used.

② This will give a plot of % Intensity as a function of RMS angle.

③ Convert to RMS angle to Table angle using the fit eq'n from previous steps. Truncate data, "high" table angles are not valid λ's. A horizontal shift may be needed. Peaks are centered roughly @ 56° when properly scanned

④ Convert Table angle to λ [nm] using the equations in Appendix 1 (Equations A4 & A5 used for this)

⑤ In xmgrace, fit one plot of %I vs. λ [nm] to get a value for the filaments' temperature

(blank)