6. The Millikan Oil Drop Experiment

6.1. Purpose:

- To determine the discreteness of the charge on the electron
- To measure the charge of an electron.

6.2. Apparatus

Welch No. 0618 Millikan apparatus, latex spheres in alcohol and water solution, stopwatch, variable voltage regulated power supply and 4W 10kilohm potentiometer, multimeter, video camera, television.

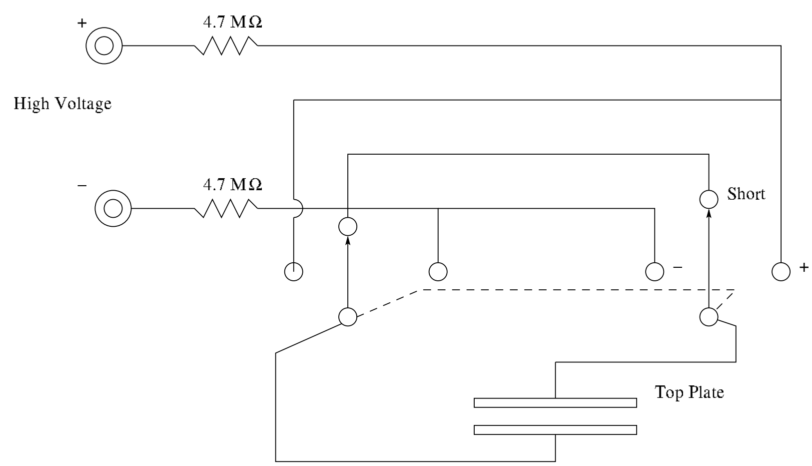

Figure 6.1.: Parallel plates and voltage supply circuit

6.3. Description and Theory

The electronic charge, or electrical charge carried by an electron, is a fundamental constant in physics. During the years 1909 1913, R. A. Millikan used the oil drop experiment to demonstrate the discreteness, or singleness of value, of the electronic charge, and to make the first accurate measurement of the value of this constant.

In that experiment, a small charged drop of oil is observed in a closed chamber between two horizontal parallel plates as shown in figure 6.2. By measuring the drop's velocity of fall and rise under gravity with the plates at a high potential difference, data can be obtained from which the charge on the drop may be computed.

The oil drops in the electric field between the plates are subject to three different forces as shown in figure 6.3 and figure 6.4: gravitational, electric and viscous. By analyzing these various forces, an

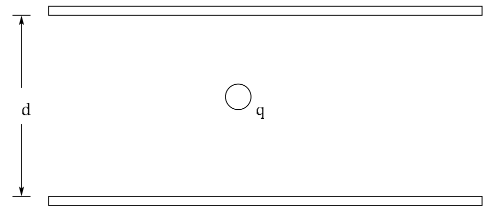

Figure 6.2.: Parallel plates separated by d with charge q. The electric field is not shown.

expression can be derived which will enable measurement of the charge on the oil drop and determination of the unit charge on the electron.

If m is the mass of the drop (sphere) under observation between the plates the gravitational force on the drop is $F_g = mg$. If there is no electrical field between the plates the drop will fall slowly, very quickly reaching a constant or terminal velocity due to the viscous retarding force. This force is $F_a = Krv_g$ (from Stoke's Law) where K is a constant for a given fluid, r is the radius of the sphere, and v_g is the terminal velocity. When the sphere has gained its terminal velocity $F_g = F_a$ and

$$mg = Krv_g \tag{6.1}$$

The size of oil drops varies and must be determined indirectly. Since the mass of an oil drop is size dependent it must also be determined indirectly.

The apparatus used for this experiment makes use latex spheres in place of oil drops. The spheres are of known fixed size and density which eliminates these complications and greatly simplifies the experimental procedures. Latex spheres are of uniform size except for occasional fragments and occasional clusters of two or more. Fragments and clusters can be quickly distinguished by visually by observing their free fall. Fragments fall slowly compared with most spheres while clusters fall more rapidly than most spheres. Under high voltage, fragments have very little difference between their upward and downward velocities. Clumps are the opposite: their upward and downward velocities are very different. Clumps may also appear to be a brighter object.

Hereafter this discussion is with reference to latex spheres. Since r is a constant, equation 6.1 becomes

$$mg = Cv_g \tag{6.2}$$

where Kr = constant = C.

When an electric field E is applied between the plates in such a direction as to make the latex spheres move upward with a constant velocity v_E, the electrical force on the sphere is $F_E = Eq$, q being the charge on the sphere. When the sphere reaches constant velocity, v_E, the forces are in equilibrium and

$$F_E - F_g = F_a \tag{6.3}$$

or

$$Eq - mg = Cv_E. \tag{6.4}$$

Solving equation 6.4 for mg and equating with equation 6.2,

$$q = \frac{C}{E}\left(v_E + v_g\right). \tag{6.5}$$

For all latex spheres of the same size and with a constant electric field a change in q results only in a change in v_E and

$$\Delta q = \frac{C}{E}\Delta v_E. \tag{6.6}$$

When many values of Δv_E are obtained, it is found that they are always integral multiples of a certain small value. Since this is true for Δv_E, the same must be true for Δq; that is, the change gained or lost is the exact multiple of some small charge. Thus the discreteness of charge may be demonstrated without actually obtaining a numerical value of the charge.

If the electric field E is varied until the electric force F_E on the sphere equals the gravitational force F_g, the sphere will remain stationary. Since there is no movement, the viscous force is zero. Therefore

$$F_g = F_E \text{ or } mg = qE \tag{6.7}$$

and

$$q = \frac{mg}{E} \tag{6.8}$$

Or since $E = \frac{V}{d}$ where d is the distance between plates and V is the voltage across the plates,

$$q = \frac{mgd}{V} \tag{6.9}$$

The mass m of a sphere is unchanging and the same for all spheres and may be computed from the radius and density specified on the bottle. The value of d is known. The value of V, the voltage across the plates can be measured with a voltmeter. Substituting these values and the value of g in equation 6.9 the quantity of charge q on the sphere can be calculated it is important to understand that this is not necessarily the value of the charge on an electron.

Note that the term mgd is a constant and therefore q is proportional to V^{-1}. When many observations are made and many values of q have been calculated, analysis of the data and results will show that values of q are always integral multiples of some small value. This small value is the fundamental unit of charge or charge on the electron. Thus electrical charge has been shown to be quantized.

If we observe the action of the spheres under free fall it is seen that some fall very fast, others extremely slow indicating the presence of clusters and fragments. it is desirable to eliminate all but single particles of essentially the same size. Clusters and fragments are referred to as 'bad masses'. These data points are eliminated by examining a bar graph of the difference in velocities. This results from examination of equation 6.15. With the electric field running from bottom to top, the forces are:

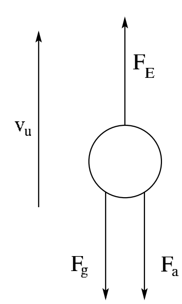

Figure 6.3.: Sphere rising

Where $F_E = qE$, the electric force

$F_g = mg$, the weight

$F_a = cv_u$, viscous force

At equilibrium, $F_E = F_g + F_a$ and the drop travels at terminal velocity v_u.

$$F_E - F_g = F_a = cv_u \tag{6.10}$$

With the electric field running from top to bottom, the forces are:

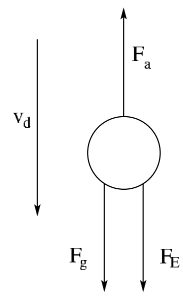

Figure 6.4.: Sphere falling

At equilibrium

$$F_E + F_g = F_a$$

and the drop travels at terminal velocity v_d.

$$F_E + F_g = F_a = cv_d \tag{6.11}$$

Hence, adding equations (6.10) and (6.11) gives

$$2F_E = c(v_u + v_d) \tag{6.12}$$

and moving the 2 over:

$$F_E = \frac{c}{2}(v_u + v_d) = qE \tag{6.13}$$

Subtracting equations (6.10) and (6.11) gives

$$2F_g = c(v_d - v_u) \tag{6.14}$$

In all cases v_d will be numerically greater than v_u. Moving the two over:

$$F_g = \frac{c}{2}(v_d - v_u) = mg \tag{6.15}$$

Equation 6.15 indicates that values of $(v_d - v_u)$ for a set of drops will group according to the weight, and therefore, the mass of the drop, for the same electric field. Thus if you analyze the values of $(v_d - v_u)$ you should be able to see certain values of mass showing up, i.e. one sphere, a cluster of two, a fragment, etc.

On the other hand, for drops of the same size and mass, equation 6.13 indicates that values of $v + v_u$ will be a measure of the electric force on the set of drops, and if the electric field has been kept constant, the data will show certain values indicating one, two, or more units of electronic charge. In the analysis you will look for values of $v_d + v_u$ that are multiples of a smallest value.

6.3.1. Notes and Comments

- It has been observed that spheres with a diameter of l.0 microns have a free fall time 12 14 seconds per reticle division and best results are obtained and much time saved if only those spheres are selected
- The results are more meaningful if spheres with relatively few electrons are selected for use in obtaining data. The use of 200 volts or more across the plates directs the experimenter to those spheres having few electrons since those with a large number of electrons move too fast in a high electric field to be timed. This is further aided by choosing slowly moving spheres.
- One of the principal reasons for difficulty in seeing particles is that the microscope is not focused properly. Focus adjustment is done using the dial with the black knob.

6.4. Testing the Apparatus

1. Connect the 6.3v supply to the blue binding posts, marked 6.3 volts.
2. Connect the high voltage supply to the red and black binding posts, observing polarity.
3. Set the reversal switch to its central position, or shorting position.
4. Turn the power supply ON. The lamp should light.
5. Turn the high voltage control up to approximately
   a) 195 to 205 volts for the blue apparatus
   b) 230 to 240 volts for the black apparatus
6. Squeeze the rubber bulb a few times thereby spraying some latex spheres into the viewing chamber. It may be necessary to place your finger over the hole in the bulb to force the air and spheres into the chamber. When properly focused, the small spheres appear as bright points of light against a dark background.
7. Move the polarity switch to apply the voltage to the plates and observe the effect on the particles. Most of the spheres become charged by friction during the spraying process. Some particles move rapidly upward and others downward indicating that there are both negatively and positively charged particles. The highly charged particles move more quickly than those with a small charge and quickly disappear from the field of view.
8. Reverse the switch position and note the effect. Also vary the electric field by means of the voltage control knob and note the effect.
9. Clumps of particles consisting of several particles clinging together fall[1] more rapidly than single particles. Fragments fall more slowly than whole particles. Attempt to avoid clumps and fragments, although neither of these are easy to discern just by observation.
10. Select a single particle that is *moving slowly in the electric field* and try to make it remain motionless by carefully adjusting the voltage. This will be recorded as the holding voltage V_H. Remember that the smaller the charge, the higher the voltage required to hold the particle stationary.

6.5. Experimental Procedure

In all laboratory work with the Millikan apparatus it is best for two students to work together. They should take turns making observations and recording data, one person observing the spheres, using the stop watch and adjusting the voltage. The other student should read the stop watch and record the times.

Use the apparatus to collect data for at least twenty particles (select those moving slowly in the electric field since these will likely be single drops with small multiples of electronic charge) and measure pairs of velocity values of v_u and v_d and voltage V_{hold} *for the same sphere.*

If time permits, attempt to time the same particle moving in both directions a second time. This can give you an idea of the error in velocity for a sphere. This may difficult because the drops tend to drift out of the small depth of field of the microscope. Patience will be required. At first you are really training yourself to use this apparatus.

1. Record your apparatus's plate separation $d =$ $\pm$
2. Use the same high voltage you selected for every velocity trial. Make sure you always use the same value when making velocity measurements in both up and down directions.

[1]The blue apparatus inverts up and down.

Sphere	t_u [s]	d_u[mm]	$v_u[\frac{m}{s}]$	t_d [s]	d_d [mm]	$v_d[\frac{m}{s}]$	V_{hold}[V]	$v_d + v_u[\frac{m}{s}]$	$v_d - v_u[\frac{m}{s}]$
1									
2									
3									
4									
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Table 6.1.: Data collection table. Velocities are calculated.

3. Record the time required for the drop to traverse a number of reticle divisions in the up and down directions (real directions, remember). In order to keep timing errors down, attempt to have the drop traverse 1 to 2 millimeters.

4. For the same drop make a measurement of V_{Hold} for use in determining q of the electron by adjusting the voltage so that the drop is stationary in the field of view.

5. *A suggested procedure is to measure t_u, then t_d, change V to make the drop stationary, record this value of V_{hold}, adjust V back to the original standard value and try, if possible to repeat the t_u and t_d values.*

Note, although the guideline suggests 20 drops, don't feel restricted to that number however it is unlikely that the results will be conclusive for a smaller data size. Twenty trials should be considered a minimum number as it is common to reject $\frac{1}{2}$ of the data.

6.6. Data Analysis

1. Calculate the velocities required to complete Table 6.1. Use table 6.2 to get the distance.

Table 6.2.: reticle division distances

color of apparatus	distance
blue	0.5mm
black	1.0mm

2. Use xmgrace to make a histogram of the values $(v_d - v_u)$. Look to see if you can find clear evidence of similar values of $(v_d - v_u)$ i.e. drops of the same weight and mass.

3. Reject data for any drop that does not seem to conform since these spheres where bad masses. This will eliminate clumps of multiple spheres (too heavy) and fragments (too light). Figure 6.5 shows an example bar graph of some invented data similar to yours. The tall and short bars indicate bad masses.

4. Now plot a second histogram of values of $(v_d + v_u)$ for only the 'good masses'. If all goes well your data should show evidence of preferred values of $(v_d + v_u)$, i.e. drops with discrete values of charge. In particular look to see if the values of $(v_d + v_u)$ appear to be multiples of a lowest value which can then be considered to be a sphere with one electronic charge.

5. From equation (6.9) q for one the electron can be found:

$$q = \frac{mgd}{V_{hold}} \tag{6.16}$$

and for spheres with n electron charges on them:

$$nq = \frac{mgd}{V_{hold}} \tag{6.17}$$

The mass of a single drop can be calculated from the following data. The manufacturer did not state uncertainties on the latex sphere diameter and density. Assume a value of ±2% for each.

- drop diameter $d_d = 1.02 \times 10^{-6} m$
- density ρ = 1.05 g cm^{-3}= 1050 kg m^3

Other relevant data:

- The distance d between the plates is written on the apparatus as it is not the same for every one.
- $g = 9.808 \pm 0.05\, \frac{m}{s^2}$

Calculate q with error for each chosen sphere keeping in mind that there may be multiple electrons on any particular sphere.

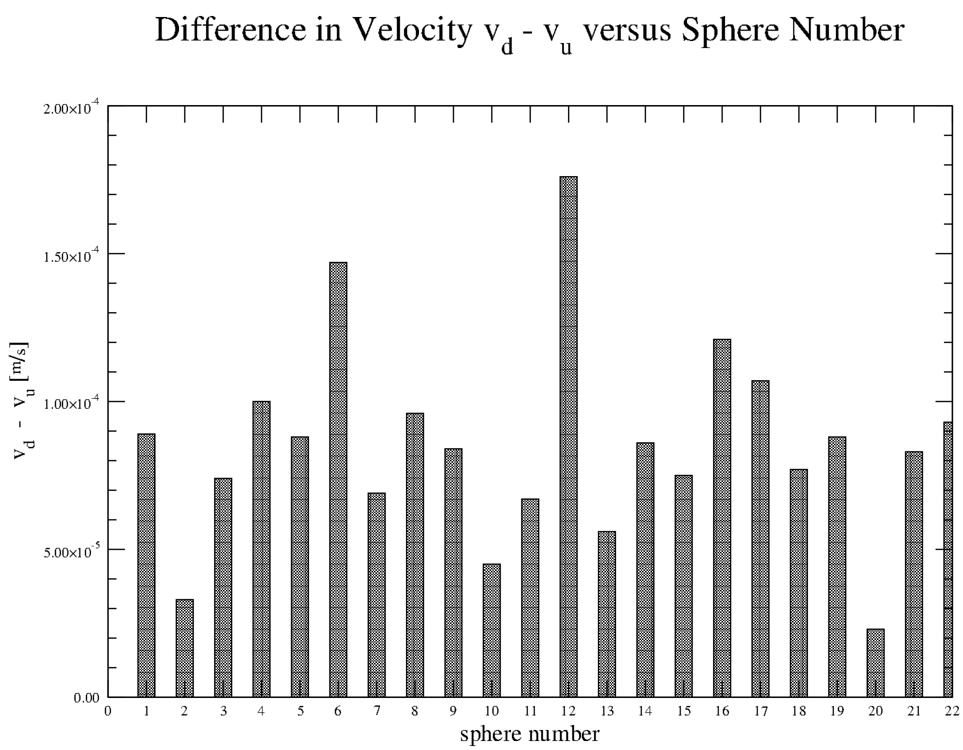

Figure 6.5.: bar graph $v_d - v_u$versus sphere number.