7. Measurement of e/m for the Electron

7.1. Introduction

At the turn of this century several crucial experiments were performed which yielded a more basic un derstanding of matter. Two of these experiments involved a detailed characterization of the electron, one experiment in 1887 by J.J. Thomson measured the ratio of the charge e of the electron to its mass m by observing the motion of an electron in electric and magnetic fields. The second experiment, by R.A. Millikan during 1909   1911, determined the charge on the electron by observing its motion in an electric field alone.

7.2. Theory

Both the above described experiments used a basic relationship in electromagnetic theory in the form of the electrostatic force and the Lorentz force on a charge q as shown below

$$\overrightarrow{F} = q(\overrightarrow{E} + \overrightarrow{v} \times \overrightarrow{B}) \tag{7.1}$$

Here q could be positive or negative depending on the charge involved. This experiment will determine the electron's ratio of charge to its mass. This will be determined by measuring the deflection of an electron beam of known kinetic energy in a homogeneous magnetic field.

Once the electron has been accelerated (and moving at constant speed) $\overrightarrow{E}$ of equation 7.1 is zero. Also the path of the electron beam is normal to the direction of the magnetic field $\overrightarrow{B}$, the $\sin\theta = 1$ in the cross product term. Since the magnetic field is fairly uniform, the electrons will follow a circular path of radius r. They will travel with a constant speed v as determined by the accelerating voltage V applied to the cathode. Since the Lorentz force is the centripetal force,

$$F_{centripetal} = evB = \frac{mv^2}{r} \tag{7.2}$$

The energy equation relating the speed of an electron accelerated by a voltage V is:

$$\frac{1}{2}mv^2 = eV \tag{7.3}$$

Figure 7.1.: e/m ratio equipment

Combining equations 7.2 and 7.3, the quantity e/m can be calculated as a function of V, r, and the magnetic field $\vec{B}$

From equation 7.2

$$Br = \frac{m}{e}v \tag{7.4}$$

and from equation 7.3

$$v = \sqrt{\frac{2eV}{m}} \tag{7.5}$$

Since the speed of the electrons will not be measured, substituting this for the velocity in equation 7.4, we find $Br = \sqrt{\frac{2mV}{e}}$ and finally

$$\frac{e}{m} = \frac{2V}{B^2r^2} \tag{7.6}$$

Although some 10^{16} electrons are present in the gas focused electron beam, these equations which are derived for one electron can be used. The electrons all have the same properties and therefore cannot be distinguished from one another. Hence, the property of a single electron can be inferred from the behavior of the number of electrons.

In this experiment, the e/m ratio as shown in equation 7.6 will be determined for the electron. If the charge on an electron is known, then this can be used to determine the mass m of an electron.
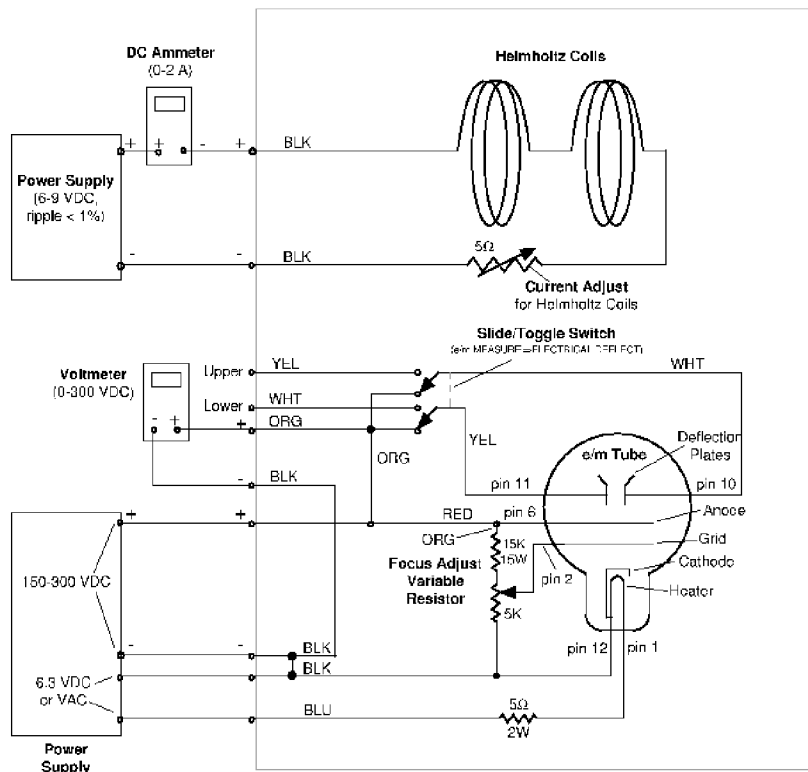
7.3. Apparatus Description

Figure 7.2.: Schematic of a similar Pasco e/m apparatus. See Table 7.1 for details on the two different manufacturers. Focus adjustment is not present on the Pacific Science or the Narika apparatus.

The equipment has been designed after the experiment of Bainbridge. It consists of a large vacuum

Table 7.2.: e/m apparatus parameters

	green Pacific supply apparatus (enclosed in wood box)	black Narika apparatus
N	*140*	*130*
a	$140mm \pm 2mm$	$150mm \pm 2mm$
recommended operating voltage	100 200 volts	160 180 volts

tube centered between a pair of Helmholtz coils[1]. An electron gun, produces a narrow beam of electrons whose paths are rendered visible by a trace[2] of helium in the tube. A schematic diagram of a similar Pasco apparatus is shown in figure 7.2.

- The accelerating voltage V_B is provided by a variable power supply B^+. A voltmeter is used to measure this voltage accurately.

- The filament current is taken from the a 6.3V AC supply. No meter required.

- The coil current is from separate 2A DC supply. Use an ammeter is in series to measure current.[3]

Table 7.1.: Required wiring for each apparatus

manufacturer	connections required
Pacific Science (wooden case)	connections for meters, coil series wire; supplies are internal
Narika (black case)	connections for external supplies and meters

7.3.1. Magnetic field B.

The magnetic field B in $\frac{weber}{m^2}$ can be calculated from the Biot Savart Law by measuring the coil current I, the coil radius a and the number of turns N in each Helmholtz coil.

$$B = \mu_0 \left(\frac{4}{5}\right)^{3/2} \frac{NI}{a} \qquad (7.7)$$

where
I is measured in amps with an error value[4].
a is the radius of the Helmholtz coils
N = number of turns in each coil. Consider this value to be exact.)
and $\mu_0 = 1.26 \times 10^{-6} \frac{weber}{amp \cdot m}$.
Consult table 7.2 for the dimensions of your apparatus.

The above equation holds for the region directly between the coils and under the condition that the thickness of the windings is negligible compared to a. The derivation of the above equations is not difficult if one begins with the expression for the magnetic field on the axis of a circular loop of wire. The field off the axis is considered in texts on electricity and magnetism. The effect of the finite thickness of the windings is discussed in the article by Bainbridge.

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[1] A pair of coils are called Helmholtz coils if they are separated by a distance equal to their radius a. This particular coil arrangement is useful because it gives an almost uniform field over a fairly large region.
[2] pressure ~10^{-2} mm Hg
[3]

or if the supply itself has sufficient resolution, a meter may not be needed.

[4] determined from meter specs and observing how steady the current is

7.4. Operation

7.4.1. Precautionary Measures

The life of the hot cathode in the cathode ray tube, like in all gas filled tubes, is limited because the positive ions caused by the cathode current, bombard and damage the cathode surface. The life of the hot cathode can be extended provided the cathode current is not *a)* high and *b)* not left on longer than necessary for the experiment. *Keeping the accelerating voltage under 230V$_{DC}$ is recommended.*

- Turn all voltage and current dials to zero before switch on. Do the same before switching off.

- When a circular beam has been created for measurement, make the diameter measurement and turn the beam off again. Alternatively, make your series of measurement reasonably quickly and turn the beam off when complete.

- Narika apparatus: The Heathkit HV supply is built with tube components. Allow the unit to warm up for five minutes before moving the DC switch from Stand by to on.

- Narika apparatus: Connect the filament lines to the e/m tube on the Narika apparatus so that its filament can warm up with the Heathkit supply warm up.

Your demonstrator will show you how to use the apparatus. At an anode voltage of around 110 to 150 volts a faint bluish glow will appear. When the room reasonably dark, the electrons first form a very faint cloud around the cathode area. When the anode voltage is increased more, the beam becomes more intense with a sharp directivity. At high voltages this directivity lessens, indicating that the optimum space charge has been exceeded.

7.4.2. Procedure:

1. Record the make and model of each digital multimeter you are using.

2. For a determination of e/m we must measure

    a) the path radius r for a known value of accelerating voltage V_B, and

    b) the value of the magnetic field B. This requires the measurement of current I.

3. Set and hold the accelerating voltage V constant. Record this value with error.

4. Make at least eight measurements varying I. Record the current values with error and the radius of the electron path for each. Assign a reasonable error to the radius values.

5. For each value of current, calculate B with error using equation 7.7.

6. Plot r^2 as a function of $\frac{1}{B^2}$ with error bars on the data points.

7. Use the slope to get a value of the $\frac{e}{m}$ ratio with error.

7.5. Question

1. Why does a lower accelerating voltage help extend the cathode's lifetime?