BRANDON UNIVERSITY

PHYSICS & ASTRONOMY

Errors - Quick Reference Sheet

<u>Addition Rules</u> $z = x + y$

Standard error (repeated measurements): $\delta z = \sqrt{\delta x^2 + \delta y^2}$

<u>Subtraction Rules</u> $z = x - y$

Standard error (repeated measurements): $\delta z = \sqrt{\delta x^2 + \delta y^2}$

<u>Multiplication Rules</u>

Multiply by constant: $z = kx$

Limit & Standard error: $\delta z = |k|\, \delta x$

EXAMPLE: $z = x + y - 3t$

Standard error: $\delta z = \sqrt{\delta x^2 + \delta y^2 + (3\delta t)^2}$

Multiply two variables: $z = xy$

Standard error: $\delta z = |z|\sqrt{\left(\frac{\delta x}{|x|}\right)^2 + \left(\frac{\delta y}{|y|}\right)^2}$

<u>Division Rules</u>

Divide two variables: $z = \frac{x}{y}$

Standard error: $\delta z = |z|\sqrt{\left(\frac{\delta x}{|x|}\right)^2 + \left(\frac{\delta y}{|y|}\right)^2}$

<u>Power Law Rules</u>

Exact power: $z = x^n$

Limit & Standard error: $\delta z = |z|\left(|n|\frac{\delta x}{|x|}\right) = |n|\, x^{n-1}\, \delta x$

EXAMPLE: $z = \frac{xy}{t^3}$

Standard error: $\delta z = |z|\sqrt{\left(\frac{\delta x}{x}\right)^2 + \left(\frac{\delta y}{y}\right)^2 + \left(\frac{3\delta t}{t}\right)^2}$

Philosophy of Experimental Physics: ***any*** measured quantity is <u>completely meaningless</u> without an uncertainty!

Special Function Rules

Function of one variable $z = f(x)$

Limit & Standard error: $\delta z = \left|\frac{df}{dx}\right| \delta x$

or numerically: δz = maximum of $|f(x+\delta x) - f(x)|$, $|f(x-\delta x) - f(x)|$

EXAMPLES:

$z = \sin x$	$\delta z = \lvert\cos x\rvert\, \delta x$ note: δx in *radians*
$z = \cos x$	$\delta z = \lvert\sin x\rvert\, \delta x$
$z = e^x$	$\delta z = \lvert e^x\rvert\, \delta x$
$z = \ell n\, x$	$\delta z = \frac{\delta x}{\lvert x\rvert}$
$z = \sqrt{x} = x^{\frac{1}{2}}$	$\delta z = \frac{\delta x}{2\sqrt{x}}$

Functions of two variables $z = f(x, y)$

Standard error: $\delta z = \sqrt{\left(\frac{\partial f}{\partial x}\delta x\right)^2 + \left(\frac{\partial f}{\partial y}\delta y\right)^2}$

EXAMPLE: thin lens equation $\frac{1}{f} = \frac{1}{s_{\text{object}}} + \frac{1}{s_{\text{image}}}$

where $s_o = 100.0 \pm 0.5$ cm
$s_i = 10.0 \pm 0.5$ cm

and thus $f = \frac{s_{\text{object}}\, s_{\text{image}}}{s_{\text{object}} + s_{\text{image}}} = \frac{100.0 \times 10.0\ \text{cm}^2}{100.0\text{cm} + 10.0\text{cm}} = 9.1$ cm

First find your partial derivatives $\frac{\partial f}{\partial s_0} = \frac{s_i}{s_o + s_i} - \frac{s_o s_i}{(s_o + s_i)^2}$ or simply $= \frac{s_i - f}{s_o + s_i}$

and $\frac{\partial f}{\partial s_i} = \frac{s_o}{s_o + s_i} - \frac{s_o s_i}{(s_o + s_i)^2}$ or $= \frac{s_o - f}{s_o + s_i}$

Use formula $\delta f = \sqrt{\left(\frac{\partial f}{\partial s_o}\delta s_o\right)^2 + \left(\frac{\partial f}{\partial s_i}\delta s_i\right)^2}$

$$= \sqrt{\left(0.5\text{cm}\frac{(10.0\text{cm} - 9.1\text{cm})}{(100\text{cm} + 10\text{cm})}\right)^2 + \left(0.5\text{cm}\frac{(100.0\text{cm} - 9.1\text{cm})}{(100\text{cm} + 10\text{cm})}\right)^2} = 0.4\ \text{cm}$$

and hence $f = 9.1 \pm 0.4$ cm

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